NAVAL POSTGRADUATE SCHOOL
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A SIMULATION STUDY OF ESTIMATES
OF A FIRST PASSAGE TIME DISTRIBUTION
FOR A SEMI-MARKOV PROCESS

by

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March 1987

Thesis Advisor P. A. Jacobs

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by

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ABSTRACT

This thesis reports on a simulation study of parametric and nonparametric procedures for obtaining confidence intervals for the logarithm of the probability a semi-markov process enters a particular state before a fixed time t. Three estimators and confidence interval procedures are proposed and compared. The different estimators use different amounts of information about the process. The maximum likelihood estimator and its normal confidence interval procedure uses the most; the estimator based on the empirical distribution function of the observed first passage times uses the least. An estimator based on an exponential approximation to the survivor function of the first passage time uses an intermediate amount of information; confidence intervals for the last estimator are obtained using jackknife and bootstrap procedures. The maximum likelihood procedure is the most efficient if the underlying model is correct. If the model is not correct the empirical survivor function estimator appears to be best for small times and the estimator based on the exponential approximation best for large times.

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I. INTRODUCTION

A. OBJECTIVES

Finite state space semi-Markov models find application in a variety of areas such as queueing theory, reliability, and clinical trials. The application of these models often centers on the distribution of a *first-passage time* to a state or a set of states representing for example the lifetime of a system or marking the end of a busy period for a server. Suppose that the *observations* of the path of the semi-Markov process are all that is known about the process.

This thesis reports the results of a simulation experiment to compare various parametric and nonparametric estimators of the natural logarithm of the probability a semi-Markov process does not enter a particular state before a given fixed time t. In what follows we will use "ln probability" for " natural logarithm of the probability ". The specific semi-Markov model and estimators considered are given in Chapter II. Chapter III contains the details of the simulation experiment and results. Conclusions from the study are given in Chapter IV.

B. SCOPE OF THE THESIS

The purpose of this thesis is to use simulation to compare estimators and their confidence intervals for the ln probability a particular semi-Markov process does not enter a particular state before a given fixed time t. The particular semi-Markov model and estimators considered are given in Chapter II. A simulation study comparing bias and standard errors for these estimators was reported in Gallagher (1986) [Ref. 1]. In this thesis, we are primarily interested in comparing confidence interval procedures.

An estimation procedure which uses the least information about the semi-Markov process uses the empirical distribution of the observed first passage times. An estimation procedure which uses the most information is to assume a parametric form for the sojourn time distribution in each state and a transition matrix to describe the transition between states; the parameters of the sojourn time distribution and transition probabilities can be estimated using maximum likelihood. An estimation procedure requiring less information uses nonparametric estimators of the sojourn time distributions and the maximum likelihood estimators for the transition probabilities.

In many cases, parametric assumptions concerning the sojourn time distribution are difficult to justify. It was demonstrated in Park (1986) [Ref. 2] that incorrect parametric assumptions may lead to very biased estimators. Hence, a nonparametric estimation procedure may be preferred to a parametric one when actual data is used. However, the nonparametric procedure can be expected to be less efficient than a parametric one provided the parametric model is correct.

The thesis is organized as follows. In Chapter II, the nature of the problem is described and several parametric and nonparametric confidence interval estimation procedure are introduced. In Chapter III, the simulation experiment is described and results are given. Conclusions drawn from the simulation study are given in Chapter IV.

II. NATURE OF THE PROBLEM

A. DESCRIPTION OF PROBLEM

The semi-Markov process model used in the simulations to compare the estimators is as follows.

Suppose we observe N individuals. Let X_t (i) be the state of the i-th individual at time t. We will assume $\{X_t (i), t \geq 0\}$ i = 1, 2,...., N, are independent semi-Markov processes with three states { 0, 1, 2 } having the same distribution as $\{X_t, t \geq 0\}$. All individuals start at t=0 in state 1, (X_0 = 1). An individual stays in state 1 for a random length of time having distribution function F_1. Upon leaving state 1, the process transitions to state 0 with probability θ and to state 2 with probability 1-θ. If the process transitions to state 2, it spends a random length of time there having distribution function F_2. From state 2, the process transitions to state 1 with probability 1. State 0 is an absorbing state. Once in it, the individual never leaves.

For all individuals, the entire path of transitions and sojourn times are observed until the time of absorption in state 0 (Fig. 2.1).

Let

$$D = \inf \{t \geq 0, X_t = 0\},$$

be the entrance time to state 0 (or the time of death).

The problem is to estimate the logarithm of the first passage time survivor function P {D > t} for fixed time t from the data obtained by observing the N individuals.

B. THEORETICAL DEVELOPMENTS

Two estimators for P{D > t} will be described in this section. They are the maximum likelihood estimator and the asymptotic renewal estimator from a paper by P.A. Jacobs. [Ref. 1: p.2-6]

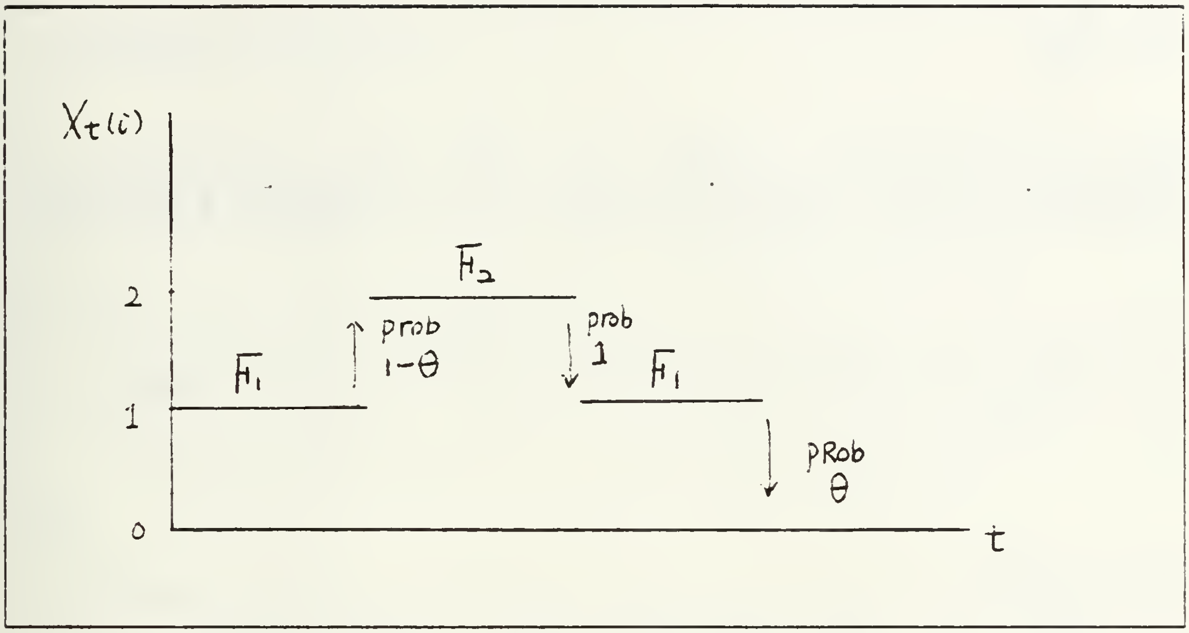

Figure 2.1 Transition State Step (Sojourn Time).

1. Maximum Likelihood Estimate for Continuous Time Markov Chain

In this subsection, the maximum likelihood estimator for $P\{D > t\}$ will be given for the special case in which the sojourn time in state i is exponentially distributed with mean $1/\rho_i$ ($i = 1, 2$).

Let R be the number of transitions from 1 to 2 for one individual. The log likelihood function for the individual is

$$L = R \ln (1-\theta) + \ln \theta + R \ln \rho_2 \qquad \text{(eqn 2.1)}$$

$$+ (1 + R) \ln \rho_1 - \rho_1 T_1 - \rho_2 T_2$$

where T_i ($i = 1,2$) is the total time spent in state i before death .

The maximum likelihood estimators are

$$\hat{\theta} = \frac{1}{1 + R} \qquad \text{(eqn 2.2)}$$

$$\hat{\rho}_1 = \frac{1 + R}{T_1} \qquad \text{(eqn 2.3)}$$

$$\hat{\rho}_2 = \frac{R}{T_2} \qquad \text{(eqn 2.4)}$$

Further R is geometric $\{\theta\}$ so $E\{R\} = (1 - \theta)/\theta$. Since the N individuals are independent, the maximum likelihood estimators using the data for all N individuals are

$$\hat{\theta} = \frac{N}{N + \tilde{R}} \qquad \text{(eqn 2.5)}$$

$$\hat{\rho}_1 = \frac{N + \tilde{R}}{\tilde{T}_1} \qquad \text{(eqn 2.6)}$$

$$\hat{\rho}_2 = \frac{\tilde{R}}{\tilde{T}_2} \qquad \text{(eqn 2.7)}$$

where $\tilde{R}$ is the total number of transitions from 1 to 2 for all N individuals and $\tilde{T}_i$ is the total time spent in state i for the N individuals. For the estimators based on data for all N individuals.

$$\text{Var}\,\{\hat{\theta}\} = I(\theta)^{-1} \qquad \text{(eqn 2.8)}$$

$$\text{Var}\,\{\hat{\rho}_1\} = I(\rho_1)^{-1} \qquad \text{(eqn 2.9)}$$

$$\text{var}\,\{\hat{\rho}_2\} = I(\rho_2)^{-1} \qquad \text{(eqn 2.10)}$$

where

$$I(\theta) = N\,E\,\{\frac{1}{\theta^2} + \frac{R}{(1-\theta)^2}\}. \qquad \text{(eqn 2.11)}$$

$$I(\rho_1) = N\,E\,\{\frac{1 + R}{\rho_1^{2}}\} \qquad \text{(eqn 2.12)}$$

$$I(\rho_2) = N\,E\,\{\frac{1}{\rho_2^{2}}\} \qquad \text{(eqn 2.13)}$$

with N being the number of individuals.

Let D be the time of entrance to state 0 for an individual. Fix t and put

$S = S(t) = P\,\{D > t\};$

then

$$S(t) = \{\frac{\lambda_2 + \rho_2}{\lambda_2} \mathrm{EXP}\,(\lambda_2\, t) - \mathrm{EXP}(\lambda_1\, t)\frac{\lambda_1 + \rho_2}{\lambda_1}\}\frac{\theta\rho_1}{\lambda_1 - \lambda_2} \qquad \text{(eqn 2.14)}$$

where λ_1, λ_2 are the roots of the equation

$$\theta\rho_1\rho_2 + y\,(\rho_1 + \rho_2) + y^2 = 0\,. \qquad \text{(eqn 2.15)}$$

A parametric estimator for S(t), the survival function, is

$$\hat{P}_M\,\{D > t\} = \{\frac{\hat{\lambda}_2 + \hat{\rho}_2}{\hat{\lambda}_2}\mathrm{EXP}(\hat{\lambda}_2 t) - \mathrm{EXP}(\hat{\lambda}_1 t)\frac{\hat{\lambda}_1 + \hat{\rho}_2}{\hat{\lambda}_1}\}\frac{\hat{\theta}\hat{\rho}_1}{\hat{\lambda}_1 - \hat{\lambda}_2} \qquad \text{(eqn 2.16)}$$

where λ_1 and λ_2 are roots of the equation

$$\hat{\theta}\hat{\rho}_1\hat{\rho}_2 + y\,(\hat{\rho}_1 + \hat{\rho}_2) + y^2 = 0\,. \qquad \text{(eqn 2.17)}$$

Since the maximum likelihood estimators are uncorrelated, the asymptotic variance of $\hat{P}_M\,\{D > t\}$, which will be denoted by $\hat{\mathrm{Var}}_M$, is approximately

$$\hat{\mathrm{Var}}_M\,\{\hat{P}_M\,\{D > t\}|\theta,\rho_1,\rho_2\} \qquad \text{(eqn 2.18)}$$

$$= \mathrm{Var}\,\{\hat{\theta}\}\,(\partial S/\partial\theta)^2 + \mathrm{Var}\,\{\hat{\rho}_1\}\,(\partial S/\partial\rho_1)^2 + \mathrm{Var}\,\{\hat{\rho}_2\}\,(\partial S/\partial\rho_2)^2$$

where

$$\frac{\partial S}{\partial \Theta} = \frac{S}{\Theta} + \frac{\partial S}{\partial \lambda_1}\frac{\partial \lambda_1}{\partial \Theta} + \frac{\partial S}{\partial \lambda_2}\frac{\partial \lambda_2}{\partial \Theta} \qquad \text{(eqn 2.19)}$$

$$\frac{\partial S}{\partial \rho_1} = \frac{S}{\rho_1} + \frac{\partial S}{\partial \lambda_1}\frac{\partial \lambda_1}{\partial \rho_1} + \frac{\partial S}{\partial \lambda_2}\frac{\partial \lambda_2}{\partial \rho_1} \qquad \text{(eqn 2.20)}$$

$$\frac{\partial S}{\partial \rho_2} = \frac{\Theta \rho_1}{(\lambda_1 - \lambda_2)}\left[\frac{1}{\lambda_2}e^{\lambda_2 t} - \frac{1}{\lambda_1}e^{\lambda_1 t}\right] + \frac{\partial S}{\partial \lambda_1}\frac{\partial \lambda_1}{\partial \rho_2} + \frac{\partial S}{\partial \lambda_2}\frac{\partial \lambda_2}{\partial \rho_2}. \qquad \text{(eqn 2.21)}$$

$$\frac{\partial S}{\partial \lambda_1} = \frac{-S}{(\lambda_1 - \lambda_2)} + \frac{\Theta \rho_1}{(\lambda_1 - \lambda_2)}\left[\frac{\rho_2}{\lambda_1^2}e^{\lambda_1 t} - \left(\frac{\rho_2}{\lambda_1} + 1\right)t e^{\lambda_1 t}\right] \qquad \text{(eqn 2.22)}$$

$$\frac{\partial S}{\partial \lambda_2} = \frac{S}{\lambda_1 - \lambda_2} + \frac{\Theta \rho_1}{(\lambda_1 - \lambda_2)}\left[\frac{-\rho_2}{\lambda_2^2}e^{\lambda_2 t} + \left(\frac{\rho_2}{\lambda_2} + 1\right)t e^{\lambda_2 t}\right] \qquad \text{(eqn 2.23)}$$

$$\frac{\partial \lambda_1}{\partial \Theta} = \frac{-\rho_1 \rho_2}{\rho_1 + \rho_2 + 2\lambda_1} \qquad \text{(eqn 2.24)}$$

$$\frac{\partial \lambda_1}{\partial \rho_1} = \frac{-\lambda_1 - \Theta \rho_2}{\rho_1 + \rho_2 + 2\lambda_1} \qquad \text{(eqn 2.25)}$$

$$\frac{\partial \lambda_1}{\partial \rho_2} = \frac{-\lambda_1 - \Theta \rho_1}{\rho_1 + \rho_2 + 2\lambda_1} \qquad \text{(eqn 2.26)}$$

The asymptotic variance of $\ln \hat{P}_M\{D>t\} = \hat{Var}_M / (\hat{P}_M\{D>t\})^2$.

2. **Asymptotic Renewal Estimator**

This subsection describes an estimation procedure for $P\{D>t\}$, which is based on an exponential approximation to the survivor function $P\{D>t\}$. Details of the approximation which is obtained by asymptotic renewal theoretic results is given in Jacobs [Ref. 3]. The approximation improves as t becomes larger.

Let $\hat{\theta} = N/(N + R)$ be the maximum likelihood estimator of θ (equation 2.5). Let $\hat{\varphi}_i(\alpha)$ be the empirical transform of the sojourn time in state i ; that is, if $S_1(i), S_2(i), \ldots, S_{M_i}(i)$ denote all the sojourn times in state i for the N individuals, then

$$\hat{\varphi}_i(\alpha) = (1/M_i) \sum_{k=1}^{M_i} \mathrm{EXP}(\alpha \times S_k(i)). \qquad \text{(eqn 2.27)}$$

The asymptotic renewal estimator of the survival distribution [Ref. 3] is

$$\hat{P}_A\{D>t\} = (\hat{b}/\hat{\mu})\, \mathrm{EXP}(-\hat{\kappa} \times t). \qquad \text{(eqn 2.28)}$$

where $\hat{\kappa}$ is the solution to the equation

$$\hat{f}(\alpha) = (1 - \hat{\theta})\, \hat{\varphi}_1(\alpha)\, \hat{\varphi}_2(\alpha) - 1 = 0\,. \qquad \text{(eqn 2.29)}$$

$$\hat{\mu} = \frac{(1-\theta)}{M_1 \times M_2} \sum_{k=1}^{M_1} \sum_{j=1}^{M_2} \{S_k(1) + S_j(2)\}\, \mathrm{EXP}\{\hat{\kappa}(S_k(1) + S_j(2))\}. \qquad \text{(eqn 2.30)}$$

$$\hat{\varphi}_1(\hat{\kappa}) = (1/M_1) \sum_{k=1}^{M_1} \mathrm{EXP}(\hat{\kappa} \times S_k(1)). \qquad \text{(eqn 2.31)}$$

and

$$\hat{b} = (\hat{\theta}/\hat{\kappa})\; \varphi_1(\hat{\kappa}). \qquad \text{(eqn 2.32)}$$

Note that when $\alpha = 0$, the left hand side of the equation (2.29) is $-\theta < 0$. As α increases, the left hand side of the equation increases to ∞, thus there is a unique solution $\hat{\kappa}$. The solution may be found by numerical search. One possible numerical search procedure is the golden section search method.

3. **Empirical Distribution estimate**

Let $d_1, d_2, \ldots, d_N$ denote the observed times of absorption in state 0 for the N individuals. A binomial estimator for the survivor function is

$$\hat{P}_B\{D > t\} = (1/N) \sum_{k=1}^{N} 1_{(t,\infty)}(d_i) \qquad \text{(eqn 2.33)}$$

where $1_{(t,\infty)}(d_i) = 1$ if $d_i > t$,
0 otherwise.

C. CONFIDENCE INTERVAL PROCEDURE

Suppose X is a random variable whose probability law depends on an unknown parameter θ. Given a random sample of X: $x_1, x_2, \ldots, x_n$ the two statistics lower (L) and upper (U) form a $100(1 - \alpha)\%$ confidence interval for θ if

$$P\{L \leq \theta \leq U\} \geq 1 - \alpha.$$

Procedures to obtain confidence intervals for the point estimates for $\ln P\{D > t\}$ are given below.

1. **Maximum Likelihood Estimator**

The maximum likelihood estimator is asymptotically normal as the number of individuals N becomes large, [Ref. 4]. For a fixed t, the maximum likelihood confidence interval for $\ln P\{D > t\}$ is

$$\{ L , U \} = \ln \hat{P}_M\{D > t\} \pm z_{1-\alpha/2} (\sqrt{\hat{Var}_M}) / (\hat{P}_M \{D < t\}). \qquad \text{(eqn 2.34)}$$

where $\hat{Var}_M$ is obtained by using the maximum likelihood estimator of θ, ρ_1, and ρ_2 in equation (2.11) - (2.26). Confidence limits that are larger than 0 are set equal to 0.

2. Empirical Distribution of First Passage Time (Binomial and Normal Approx. Confidence Interval Methods)

a. *Binomial Confidence Interval*

Since the N individuals are assumed independent, the estimator $\hat{P}_B\{D>t\}$ of equation (2.33) has a binomial distribution with N trials and probability of success $P\{D>t\}$. The IMSL routine BELBIN was used to obtain binomial confidence intervals for $P\{D>t\}$. For a description of the procedure see [Ref. 5: p.390 - 391]. The binomial confidence interval for $\ln P\{D>t\}$ is obtained by taking logarithms of the upper and lower confidence limits. If the lower confidence limits for $\hat{P}_B\{D>t\}$ is equal to 0, it is set equal to 0.0001.

b. *Approximate Normal Confidence Interval.*

If the number of individuals N is large, the binomial confidence interval procedure can be approximated by a normal confidence interval procedure as follows, [Ref. 6: p.99 - 100 , p.954 - 955]. The interval is

$$\{ L , U \} = \hat{P}_B\{D>t\} \pm z_{1-\alpha/2}\sqrt{\hat{V}ar_B} \qquad \text{(eqn 2.35)}$$

where

$$\hat{V}ar_B = (1 / N)\{ \hat{P}_B\{D>t\} \times (1 - \hat{P}_B\{D>t\}) \}$$

This interval $\{ L , U \}$ is an approximate $100(1-\alpha)\%$ confidence interval for $P\{D>t\}$. The interval for $\ln P\{D>t\}$ is $\{ \ln L , \ln U \}$. If either L or U is less than 0, it is set equal to 0.0001. If either L or U is greater than 1, it is set equal to 1.

3. Asymptotic Renewal Procedure

In this subsection, two nonparametric methods are described for obtaining confidence interval using the estimator $\hat{P}_A\{D>t\}$. One is the jackknife. The other is the bootstrap. They are described below.

a. *Jackknife Estimation Method*

The jackknife was first introduced by Quenouille (1956) [Ref. 7], for the purpose of reducing the estimate bias, and the procedure was later utilized by Tukey (1958) [Ref. 8], to develop a general method for obtaining approximate confidence intervals.

The basic idea of the jackknife estimation method is to assess the effect of each of the groups into which the data have been divided, not by the results for that

group alone, but rather through the effect upon the body of data that results from omitting that group. The two bases of the jackknife are that we make the desired calculation for all the data, and then, after dividing the data into groups, we make the calculations for each of the slightly reduced bodies of data obtained by leaving out just one of the groups. [Ref. 9]

The jackknife procedure is as follows:

(1) Let $Y_{all}(t)$ be the estimate $\hat{P}_A\{D > t\}$ computed using all the data.

(2) Let $Y_j(t)$ be the computed statistic using that data which omits the j^{th} subgroup where j = 1, 2, ..., k. In this thesis, for the case in which the number of individuals N equals 20 the j^{th} subgroup consists of all data corresponding to the j^{th} individual. For the case in which the number of individuals N equals 50 the first subgroup consists of all data corresponding to the first 5 individuals, the second subgroup of the second 5 individuals etc. Some cases for N = 50 were run with the j^{th} subgroup consisting of the j^{th} individual. The resulting confidence intervals differed little from those obtained by leaving out 5 individuals at time. Computational considerations lead us to use the fewer number of subgroups in this case.

(3) Define the j^{th} pseudo-value by

$$Y_{*j}(t) = k \ln Y_{all}(t) - (k-1) \ln Y_j(t). \qquad \text{(eqn 2.36)}$$

(4) The jackknife estimator $Y_*(t)$ is

$$Y_*(t) = (1/k)\{Y_{*_1}(t) + Y_{*_2}(t) + \dots + Y_{*_k}(t)\}. \qquad \text{(eqn 2.37)}$$

(5) The jackknife estimator of the variance of $Y_*(t)$ is

$$S_*^2(t) = \{1/(k(k-1))\} \sum_{j=1}^{k} (Y_{*j}(t) - Y_*(t))^2. \qquad \text{(eqn 2.38)}$$

Tukey(1958) proposes that in a wide variety of problems the k estimated pseudo-values can be treated as approximately independent and identically distributed random variables [Ref. 8], to obtain the following confidence interval procedure.

(6) The jackknife confidence interval is computed as follows.

$$\{ \ L , U \} = Y_*(t) \pm t_{1-\alpha/2} \sqrt{S_*^2(t)} \qquad \text{(eqn 2.39)}$$

where $t_{1-\alpha/2}$ is the upper 1- $\alpha/2$ critical point of the t-distribution with k-1 degrees of freedom. The confidence interval given by equation (2.39) is a function of the estimated variance. If either confidence limit is greater than 0, it is set equal to 0.

b. Bootstrap Estimation Method

Efron(1979) introduced the bootstrap method for estimating the distribution of a statistic computed from observations, [Ref. 10]. In this thesis, the bootstrap was implemented as follows.

Suppose data are gathered for N individuals. Let $R_{12}(n)$ be the number of transitions from state 1 to state 2 for individual n. Let $\{S_k(i)\}$ be the collection of all sojourn times in state i for all individuals. A bootstrap replication is generated as follows: To generate data for one individual, one observation is drawn at random with replacement from $\{R_{12}(n)\}$; call the observation r_{12}; $r_{12}+1$ observations are drawn at random with replacement from the collection of state 1 sojourn times $\{S_k(1)\}$; r_{12} observations are drawn at random with replacement from the collection of sojourn times in state 2, $\{S_k(2)\}$. This procedure is replicated N times to generate bootstrap data for N individuals. The estimator $\ln \hat{P}_A \{D > t\}$ is computed using the generated data. This completes one bootstrap replication; B bootstrap replications are done. The B estimates of $\ln \hat{P}_A \{D > t\}$ are ordered. A $100 \times (1-\alpha)\%$ confidence interval is constructed using the $\alpha/2$ and $1 - \alpha/2$ quantiles of the bootstrap estimates of $\ln \hat{P}_A \{D > t\}$. If either confidence limit is larger than 0, it is set equal to 0. In the simulations the number of bootstrap replications is B = 100.

III. ANALYSIS OF THE CONFIDENCE INTERVAL PROCEDURES

A. DESCRIPTION OF SIMULATION

A Fortran program is written to simulate the semi-Markov process and compute the confidence interval. All simulations are carried out on an IBM 3033AP Computer at the Naval Postgraduate School using the LLRANDOM II Random number generating package [Ref. 13].

The data for the simulated experiments are generated as follows:

(1) An individual starts in state 1 at time 0.

(2) A random number with distribution F_1 is generated for the sojourn time in state 1.

(3) A uniform random number is generated.

If it less than θ. the process transitions to state 0 and the data for one individual is complete. If it is greater than θ, the process transitions to state 2. A random number having distribution function F_2 is generated. The procedure then returns to step 2. Data are generated for N individuals and are collected as follows:

(1) The passage times to state 0 for each individual;

(2) The sojourn times in states 1 and 2 for each individual;

(3) The number of transitions from state 1 to state 2 for each individual.

For each replication of the simulation the estimates and confidence interval in Chapter II are computed for $\ln P\{D>t\}$. Each simulation is replicated 300 times. The true $\ln P\{D>t\}$ is computed. For each procedure, the number of confidence intervals covering the true value is recorded.

The number of individuals that are too low (true $\ln P\{D>t\}>U$; the upper endpoint of the interval) and too high (true $\ln P\{D>t\}<L$; the lower endpoint of the interval) are also recorded. The average length of the confidence interval is computed as well as the standard deviation of the lengths. The standard deviation is computed by subtracting the mean length from each length, squaring the results, summing over the 300 lengths and finally divided by 299.

B. SIMULATION RESULTS FOR THE EXPONENTIAL MODEL.

In this section results will be reported for a simulation experiment in which the sojourn time distributions in both states are exponential; F_1 is exponential with mean $1/\rho_1$ and F_2 is exponential with mean $1/\rho_2$. Some true values for $P\{D > t\}$ for this case can be found in Appendix A.

For each replication nominal 80%, and 90 % confidence intervals for ln $P\{D > t\}$ for various values for t are computed using each procedure of Chapter II. The times considered are t = 0.5, 1.0, 1.5, 2.0, 3.0, 4.0.

The confidence interval procedures for ln $P\{D > t\}$ are the binomial confidence interval (= BIN), and it's approximating normal confidence interval (= NOR), for the ln fraction of individuals who have not entered state 0 by time t: the maximum likelihood confidence interval (= MLE), jackknife confidence interval for the asymptotic renewal estimator (= J.K), and bootstrap confidence interval for the asymptotic renewal estimator (= B.T). For each procedure the number of intervals covering the true value of ln P $\{D > t\}$ is recorded as well as the number of intervals that are too high or too low. These results are reported in Tables 1, 3, 5 and 7. Next to each coverage count are given the corresponding coverage proportion in parenthesis. Below the coverage numbers is given a 95% confidence intervals for the coverage rates. This interval is computed as follows. If P is the proportion of $(1-\alpha)$% intervals that cover the true value then a 95% confidence interval for the coverage rate is

$$\{ CL , CU \} = P \pm 1.96\sqrt{P(1- P)/300}.$$

If a $(1-\alpha)$% confidence interval procedure is performing well, then this interval should cover about $(1-\alpha)$% of the time. In addition, if a 80% (respectively 90%) confidence interval procedure is working well, then out of 300 replications between 226 (respectively 260) and 254 (respectively 280) confidence intervals should cover the true value. Simulations for which the numbers of intervals that cover are outside of these bounds are given in bold type. In the first column of Tables 1, 3, 5 and 7 the true $P\{D > t\}$ is given in parentheses.

In Tables 2, 4, 6 and 8 are recorded the average length of the confidence intervals for ln $P\{D > t\}$. The estimated standard deviation of the length is below the average length in parenthesis. If a procedure is performing well, it should not only have the correct coverage rate but also a small average length. The simulation results recorded in Tables 1 - 4 are for a simulation whose parameter values are $\rho_1 = 1.0$, $\rho_2 = 10.0$ and

$\theta = 0.5$. The number of individuals N is set at 20 and 50, representing a low and moderate number of individuals.

Coverage results for N = 50 individuals are presented in Table 1. The binomial confidence interval procedure tends to overcover. The maximum likelihood procedure has the right coverage for all cases. The two confidence interval procedures using the asymptotic renewal estimators undercover for $t \leq 1.0$ and have the right coverage for $t \geq 1.5$. When a jackknife confidence interval does not cover it is often because it is too high (true value < L).

Sample means and variance of the confidence interval lengths, for N = 50 individuals, are reported in Table 2. The average lengths for the maximum likelihood and bootstrap are very close for t larger than 1.0. The bootstrap and jackknife procedures have larger average confidence interval lengths than those for the maximum likelihood estimator but smaller than for the binomial procedure.

Results for a simulation with N = 20 individuals are given in Tables 3 and 4. They are similiar to those in Tables 1 and 2. However, the average length of the intervals is larger in Table 4 than in Table 2 reflecting the smaller number of individuals.

Results of a simulation with the same parameters as the first but with the $\theta =$ 2/3 are given in Tables 5 - 8. Results for N = 50 individuals are given in Tables 5 and 6. The coverage results are in Table 5. The binomial procedure has better coverage than in Table 1. The procedure based on the asymptotic renewal estimators have the correct coverage only for $t \geq 2.0$. The average length of the intervals in Table 6 are in general longer than the lengths in Table 2.

Results for a simulation with $\theta = 2/3$ and N = 20 individuals are given in Table 7 and 8. The binomial confidence interval tends to overcover when the true probability is greater than 0.5241 or less than 0.2753. Once again the confidence intervals based on the asymptotic renewal procedure undercover for $t < 2.0$. The maximum likelihood confidence intervals give the correct coverage.

TABLE 1

COVERAGE RATIO (EXP MODEL)

$N = 50, \theta = 0.5, \rho_1 = 1.0, \rho_2 = 10.0$

time	C.I (%)	coverage	BIN	NOR	MLE	J.K	B.T
0.5	80 %	too high	17(.06)	38(.13)	27(.09)	119(.40)	151(.50)
		cover	**260(.87)**	239(.80)	245(.83)	**167(.56)**	**149(.50)**
		too low	23(.08)	23(.08)	28(.09)	14(.05)	0(.00)
		int.val	(.83,.91)	(.75,.84)	(.77,.86)	(.50,.61)	(.44,.55)
(.7866)	90 %	too high	5(.02)	17(.06)	14(.05)	83(.28)	110(.37)
		cover	**288(.96)**	276(.92)	274(.91)	**209(.70)**	**190(.63)**
		too low	7(.02)	7(.02)	12(.04)	8(.03)	0(.00)
		int.val	(.94,.98)	(.89,.95)	(.88,.95)	(.64,.75)	(.58,.69)
1.0	80 %	too high	28(.09)	28(.09)	27(.09)	70(.23)	80(.27)
		cover	245(.82)	245(.82)	245(.82)	**216(.72)**	**213(.71)**
		too low	27(.09)	27(.09)	28(.09)	14(.05)	7(.02)
		int.val	(.77,.86)	(.77,.86)	(.77,.86)	(.67,.77)	(.66,.76)
(.6203)	90 %	too high	8(.03)	12(.04)	14(.05)	45(.15)	45(.15)
		cover	**282(.94)**	275(.92)	274(.91)	**247(.82)**	**250(.83)**
		too low	10(.03)	13(.04)	12(.04)	8(.03)	5(.02)
		int.val	(.91,.97)	(.89,.95)	(.88,.95)	(.78,.87)	(.79,.88)
1.5	80 %	too high	19(.06)	23(.08)	27(.09)	53(.17)	46(.15)
		cover	**262(.87)**	243(.81)	245(.82)	235(.78)	233(.78)
		too low	19(.06)	34(.11)	28(.09)	12(.04)	21(.07)
		int.val	(.84,.91)	(.77,.85)	(.77,.86)	(.74,.83)	(.73,.82)
(.4891)	90 %	too high	11(.04)	11(.04)	14(.05)	32(.11)	23(.08)
		cover	278(.93)	278(.93)	274(.91)	262(.87)	266(.89)
		too low	11(.04)	11(.04)	12(.04)	6(.02)	11(.04)
		int.val	(.89,.96)	(.90,.93)	(.88,.95)	(.84,.91)	(.85,.92)
2.0	80 %	too high	18(.06)	31(.10)	27(.09)	47(.16)	31(.10)
		cover	**260(.87)**	233(.78)	245(.82)	240(.80)	243(.81)
		too low	22(.07)	36(.12)	28(.09)	13(.04)	26(.09)
		int.val	(.83,.91)	(.73,.82)	(.77,.86)	(.75,.85)	(.77,.85)
(.3857)	90 %	too high	9(.03)	9(.03)	14(.05)	22(.07)	15(.05)
		cover	277(.92)	269(.90)	274(.91)	270(.90)	268(.89)
		too low	14(.05)	22(.07)	12(.04)	8(.03)	17(.06)
		int.val	(.89,.95)	(.86,.93)	(.88,.95)	(.87,.93)	(.86,.93)
3.0	80 %	too high	20(.07)	20(.07)	27(.09)	36(.12)	25(.08)
		cover	**262(.87)**	247(.82)	245(.82)	250(.83)	240(.80)
		too low	18(.06)	33(.11)	28(.09)	14(.05)	35(.12)
		int.val	(.84,.91)	(.78,.87)	(.77,.86)	(.79,.88)	(.75,.85)
(.2399)	90 %	too high	13(.04)	13(.04)	14(.05)	20(.07)	11(.04)
		cover	**283(.94)**	269(.90)	275(.92)	274(.91)	267(.89)
		too low	4(.01)	18(.06)	11(.04)	6(.02)	22(.07)
		int.val	(.92,.97)	(.86,.93)	(.89,.95)	(.88,.95)	(.85,.93)
4.0	80 %	too high	14(.05)	14(.05)	27(.09)	35(.12)	20(.07)
		cover	**278(.93)**	253(.84)	245(.82)	248(.83)	239(.80)
		too low	8(.03)	33(.11)	28(.09)	17(.06)	41(.14)
		int.val	(.90,.96)	(.80,.88)	(.77,.86)	(.78,.87)	(.75,.84)
(.1492)	90 %	too high	6(.02)	6(.02)	14(.05)	19(.06)	10(.03)
		cover	**286(.95)**	261(.87)	275(.92)	274(.91)	262(.87)
		too low	8(.03)	33(.11)	11(.04)	7(.02)	28(.09)
		int.val	(.93,.98)	(.83,.91)	(.89,.95)	(.88,.95)	(.84,.91)

TABLE 2

LENGTH OF C.I FOR ln P{D > t} (EXPONENTIAL MODEL)
$N = 50, \theta = 0.5, \rho_1 = 1.0, \rho_2 = 10.0$

time	C.I (%)	BIN	NOR	MLE	J.K	B.T
0.5	80 %	0.2161 (0.033)	0.1902 (0.032)	0.0889 (0.012)	0.1546 (0.080)	0.1267 (0.038)
	90 %	0.2709 (0.042)	0.2447 (0.041)	0.1141 (0.016)	0.2048 (0.106)	0.1619 (0.047)
1.0	80 %	0.3188 (0.049)	0.2882 (0.042)	0.1768 (0.024)	0.2022 (0.066)	0.1810 (0.032)
	90 %	0.4008 (0.056)	0.3717 (0.055)	0.2271 (0.032)	0.2680 (0.088)	0.2344 (0.041)
1.5	80 %	0.4164 (0.056)	0.3798 (0.053)	0.2648 (0.037)	0.2822 (0.088)	0.2623 (0.045)
	90 %	0.5237 (0.071)	0.4919 (0.079)	0.3400 (0.048)	0.3740 (0.117)	0.3398 (0.061)
2.0	80 %	0.5238 (0.082)	0.4802 (0.063)	0.3528 (0.049)	0.3773 (0.120)	0.3522 (0.065)
	90 %	0.6584 (0.100)	0.6253 (0.076)	0.4530 (0.064)	0.5001 (0.159)	0.4583 (0.086)
3.0	80 %	0.7496 (0.131)	0.6925 (0.125)	0.5287 (0.074)	0.5846 (0.188)	0.5449 (0.107)
	90 %	0.9404 (0.164)	0.9173 (0.178)	0.6790 (0.095)	0.7749 (0.249)	0.7059 (0.140)
4.0	80 %	1.0498 (0.260)	0.9973 (0.311)	0.7047 (0.099)	0.7988 (0.261)	0.7380 (0.148)
	90 %	1.3136 (0.323)	1.4207 (0.737)	0.9049 (0.127)	1.0590 (0.346)	0.9589 (0.194)

TABLE 3

COVERAGE RATIO (EXP MODEL)

$N = 20, \theta = 0.5, \rho_1 = 1.0, \rho_2 = 10.0$

time C.I (%)	coverage	BIN	NOR	MLE	J.K	B.T
0.5 80 %	too high	14(.05)	40(.13)	24(.08)	106(.35)	140(.47)
	cover	**272(.91)**	228(.76)	251(.84)	**174(.58)**	**160(.53)**
	too low	14(.05)	32(.11)	25(.08)	20(.07)	0(.00)
	int.val	(.87,.94)	(.71,.81)	(.79,.88)	(.52,.64)	(.48,.59)
(.7866) 90 %	too high	4(.01)	40(.13)	14(.05)	78(.26)	99(.33)
	cover	**282(.94)**	**246(.82)**	276(.92)	**211(.70)**	**201(.67)**
	too low	14(.05)	14(.05)	10(.03)	11(.04)	0(.00)
	int.val	(.91,.97)	(.78,.86)	(.89,.95)	(.65,.76)	(.62,.72)
1.0 80 %	too high	17(.06)	40(.13)	24(.08)	72(.24)	66(.22)
	cover	250(.83)	227(.76)	250(.83)	**210(.70)**	**224(.75)**
	too low	33(.11)	33(.11)	26(.09)	18(.06)	10(.03)
	int.val	(.79,.88)	(.71,.81)	(.79,.88)	(.65,.75)	(.70,.80)
(.6203) 90 %	too high	7(.02)	17(.06)	14(.05)	46(.15)	33(.11)
	cover	275(.92)	265(.88)	277(.92)	**245(.82)**	**259(.86)**
	too low	18(.06)	18(.06)	9(.03)	9(.03)	8(.03)
	int.val	(.88,.95)	(.85,.92)	(.89,.95)	(.77,.86)	(.82,.90)
1.5 80 %	too high	15(.05)	26(.09)	24(.08)	48(.16)	40(.13)
	cover	**265(.88)**	226(.75)	250(.83)	247(.82)	236(.79)
	too low	20(.07)	48(.16)	26(.09)	5(.02)	24(.08)
	int.val	(.85,.92)	(.70,.80)	(.79,.88)	(.78,.87)	(.74,.83)
(.4891) 90 %	too high	15(.05)	15(.05)	14(.05)	33(.11)	19(.06)
	cover	278(.93)	265(.88)	277(.92)	266(.89)	266(.89)
	too low	7(.02)	20(.07)	9(.03)	1(.00)	15(.05)
	int.val	(.88,.95)	(.85,.92)	(.89,.95)	(.85,.94)	(.85,.94)
2.0 80 %	too high	11(.04)	24(.08)	24(.08)	50(.17)	27(.09)
	cover	**267(.89)**	**220(.73)**	250(.83)	245(.83)	240(.80)
	too low	22(.07)	56(.19)	26(.09)	5(.02)	33(.11)
	int.val	(.85,.93)	(.68,.78)	(.79,.88)	(.77,.86)	(.75,.85)
(.3857) 90 %	too high	11(.04)	11(.04)	14(.05)	28(.09)	10(.03)
	cover	280(.93)	267(.89)	277(.92)	271(.90)	267(.89)
	too low	9(.03)	22(.07)	9(.03)	1(.00)	23(.08)
	int.val	(.91,.96)	(.85,.93)	(.89,.95)	(.87,.94)	(.85,.93)
3.0 80 %	too high	23(.08)	23(.08)	23(.08)	40(.13)	19(.06)
	cover	**271(.90)**	249(.83)	251(.84)	252(.84)	238(.79)
	too low	6(.02)	28(.09)	26(.09)	8(.03)	43(.14)
	int.val	(.87,.94)	(.79,.87)	(.79,.88)	(.80,.88)	(.75,.84)
(.2399) 90 %	too high	8(.03)	8(.03)	14(.05)	22(.07)	9(.03)
	cover	**286(.95)**	264(.88)	277(.92)	276(.92)	260(.87)
	too low	6(.02)	28(.09)	9(.03)	2(.01)	31(.10)
	int.val	(.93,.98)	(.84,.92)	(.89,.95)	(.89,.95)	(.83,.91)
4.0 80 %	too high	19(.06)	19(.06)	23(.08)	36(.12)	15(.05)
	cover	**271(.90)**	**224(.75)**	251(.84)	253(.84)	231(.77)
	too low	10(.03)	57(.19)	26(.09)	11(.04)	54(.18)
	int.val	(.87,.94)	(.70,.80)	(.79,.88)	(.81,.89)	(.72,.82)
(.1492) 90 %	too high	3(.01)	3(.01)	14(.05)	21(.07)	7(.03)
	cover	**287(.96)**	**240(.80)**	277(.92)	277(.92)	261(.87)
	too low	6(.02)	28(.09)	9(.03)	2(.01)	32(.11)
	int.val	(.93,.98)	(.75,.85)	(.89,.95)	(.89,.95)	(.83,.91)

TABLE 4

LENGTH OF C.I FOR ln P{D < t} (EXPONENTIAL MODEL)
$N = 20, \theta = 0.5, \rho_1 = 1.0, \rho_2 = 10.0$

time	C.I (%)	BIN	NOR	MLE	J.K	B.T
0.5	80 %	0.3696 (0.092)	0.3026 (0.092)	0.1446 (0.033)	0.2083 (0.079)	0.1854 (0.059)
	90 %	0.4598 (0.115)	0.3897 (0.122)	0.1858 (0.042)	0.2713 (0.103)	0.2449 (0.081)
1.0	80 %	0.5451 (0.131)	0.4703 (0.125)	0.2877 (0.066)	0.3072 (0.108)	0.2899 (0.081)
	90 %	0.6797 (0.164)	0.6132 (0.168)	0.3695 (0.085)	0.4001 (0.140)	0.3816 (0.110)
1.5	80 %	0.7156 (0.167)	0.6320 (0.159)	0.4308 (0.099)	0.4668 (0.180)	0.4361 (0.130)
	90 %	0.8928 (0.208)	0.8351 (0.227)	0.5533 (0.127)	0.6079 (0.235)	0.5790 (0.181)
2.0	80 %	0.9150 (0.273)	0.8328 (0.306)	0.5739 (0.132)	0.6452 (0.258)	0.5971 (0.186)
	90 %	1.1412 (0.340)	1.1847 (0.837)	0.7370 (0.169)	0.8403 (0.336)	0.7927 (0.259)
3.0	80 %	1.3762 (0.567)	1.3689 (0.878)	0.8601 (0.198)	1.0177 (0.419)	0.9324 (0.307)
	90 %	1.7105 (0.663)	2.2324 (1.773)	1.1045 (0.255)	1.3253 (0.546)	1.2427 (0.429)
4.0	80 %	2.1097 (1.197)	2.4656 (2.060)	1.1463 (0.264)	1.3966 (0.584)	1.2722 (0.426)
	90 %	2.5832 (1.308)	3.9914 (2.720)	1.4720 (0.339)	1.8188 (0.761)	1.6956 (0.596)

TABLE 5

COVERAGE RATIO (EXP MODEL)
$N = 50, \theta = 2/3, \rho_1 = 1.0, \rho_2 = 10.0$

time C.I (%)	coverage	BIN	NOR	MLE	J.K	B.T
0.5 80 %	too high	28(.09)	50(.17)	27(.09)	173(.58)	246(.82)
	cover	249(.83)	227(.76)	248(.83)	**121(.40)**	**54(.18)**
	too low	23(.08)	23(.08)	25(.08)	6(.02)	0(.00)
	int.val	(.79,.87)	(.71,.81)	(.78,.87)	(.35,.46)	(.14,.22)
(.7231) 90 %	too high	13(.04)	20(.07)	12(.04)	149(.50)	210(.70)
	cover	277(.92)	262(.87)	273(.91)	**148(.49)**	**90(.30)**
	too low	10(.03)	10(.03)	15(.05)	3(.01)	0(.00)
	int.val	(.89,.95)	(.84,.91)	(.88,.94)	(.44,.55)	(.25,.35)
1.0 80 %	too high	20(.07)	32(.11)	27(.09)	131(.44)	150(.50)
	cover	251(.83)	239(.80)	248(.83)	**160(.53)**	**146(.49)**
	too low	29(.10)	29(.10)	25(.08)	9(.03)	4(.01)
	int.val	(.79,.88)	(.75,.84)	(.78,.87)	(.48,.59)	(.43,.54)
(.5241) 90 %	too high	13(.04)	20(.07)	11(.04)	96(.32)	101(.34)
	cover	279(.93)	271(.90)	274(.91)	**200(.67)**	**198(.66)**
	too low	8(.03)	9(.03)	15(.05)	4(.01)	1(.00)
	int.val	(.90,.96)	(.87,.94)	(.88,.95)	(.61,.72)	(.61,.71)
1.5 80 %	too high	30(.10)	30(.10)	27(.09)	86(.29)	88(.29)
	cover	239(.80)	239(.80)	248(.83)	**206(.69)**	**199(.66)**
	too low	31(.10)	31(.10)	25(.08)	8(.03)	13(.04)
	int.val	(.75,.84)	(.75,.84)	(.78,.87)	(.63,.74)	(.61,.72)
(.3799) 90 %	too high	10(.03)	15(.05)	11(.04)	53(.18)	51(.17)
	cover	279(.93)	267(.89)	274(.91)	**244(.81)**	**243(.81)**
	too low	11(.04)	18(.06)	15(.05)	3(.01)	6(.02)
	int.val	(.90,.96)	(.85,.93)	(.88,.95)	(.77,.86)	(.77,.85)
2.0 80 %	too high	19(.06)	19(.06)	27(.09)	61(.20)	55(.18)
	cover	254(.85)	237(.79)	248(.83)	228(.76)	227(.76)
	too low	27(.09)	44(.15)	25(.08)	11(.04)	18(.06)
	int.val	(.81,.89)	(.74,.84)	(.78,.87)	(.71,.81)	(.71,.81)
(.2753) 90 %	too high	9(.03)	9(.03)	11(.04)	34(.11)	24(.08)
	cover	275(.92)	265(.88)	274(.91)	261(.87)	262(.87)
	too low	16(.05)	26(.09)	15(.05)	5(.02)	14(.05)
	int.val	(.89,.95)	(.85,.92)	(.88,.95)	(.83,.91)	(.84,.91)
3.0 80 %	too high	32(.11)	32(.11)	27(.09)	45(.15)	25(.08)
	cover	252(.84)	**220(.73)**	248(.83)	240(.80)	239(.80)
	too low	16(.05)	48(.16)	25(.08)	15(.05)	36(.12)
	int.val	(.80,.88)	(.68,.78)	(.78,.87)	(.75,.85)	(.75,.84)
(.1446) 90 %	too high	16(.05)	6(.02)	12(.04)	19(.06)	11(.04)
	cover	277(.92)	**246(.82)**	273(.91)	272(.91)	264(.88)
	too low	7(.02)	48(.16)	15(.05)	9(.03)	25(.08)
	int.val	(.89,.95)	(.78,.86)	(.88,.94)	(.87,.94)	(.84,.92)
4.0 80 %	too high	35(.12)	35(.12)	27(.09)	31(.10)	19(.06)
	cover	242(.81)	**196(.65)**	248(.83)	251(.84)	233(.78)
	too low	23(.08)	69(.23)	25(.08)	18(.06)	48(.16)
	int.val	(.76,.85)	(.60,.71)	(.78,.87)	(.79,.88)	(.73,.82)
(.0760) 90 %	too high	12(.04)	1(.00)	12(.04)	16(.05)	8(.03)
	cover	**285(.95)**	276(.92)	273(.91)	274(.91)	263(.88)
	too low	3(.01)	23(.08)	15(.05)	10(.03)	29(.10)
	int.val	(.93,.97)	(.89,.95)	(.88,.94)	(.88,.95)	(.84,.91)

TABLE 6

LENGTH OF C.I FOR $\ln P\{D > t\}$ (EXPONENTIAL MODEL)
$N = 50, \theta = 2/3, \rho_1 = 1.0, \rho_2 = 10.0$

time	C.I (%)	BIN	NOR	MLE	J.K	B.T
0.5	80 %	0.2536 (0.040)	0.2261 (0.038)	0.1196 (0.017)	0.2232 (0.092)	0.1914 (0.048)
	90 %	0.3184 (0.050)	0.2913 (0.049)	0.1535 (0.022)	0.2958 (0.122)	0.2506 (0.060)
1.0	80 %	0.3848 (0.056)	0.3502 (0.052)	0.2383 (0.034)	0.2618 (0.082)	0.2510 (0.048)
	90 %	0.4840 (0.070)	0.4530 (0.069)	0.3060 (0.044)	0.3470 (0.109)	0.3243 (0.064)
1.5	80 %	0.5260 (0.085)	0.4823 (0.080)	0.3570 (0.051)	0.3841 (0.134)	0.3635 (0.074)
	90 %	0.6612 (0.107)	0.6282 (0.107)	0.4584 (0.066)	0.5091 (0.178)	0.4783 (0.102)
2.0	80 %	0.6815 (0.126)	0.6284 (0.120)	0.4757 (0.068)	0.5387 (0.196)	0.4977 (0.106)
	90 %	0.8556 (0.157)	0.8281 (0.173)	0.6108 (0.088)	0.7140 (0.260)	0.6584 (0.144)
3.0	80 %	1.0966 (0.332)	1.0598 (0.468)	0.7131 (0.102)	0.8739 (0.332)	0.7867 (0.175)
	90 %	1.3718 (0.414)	1.5369 (0.930)	0.9157 (0.132)	1.1583 (0.440)	1.0428 (0.237)
4.0	80 %	1.7374 (0.773)	1.9119 (1.277)	0.9505 (0.137)	1.2188 (0.476)	1.0833 (0.248)
	90 %	2.1575 (0.901)	3.1281 (1.968)	1.2205 (0.176)	1.6155 (0.631)	1.4384 (0.336)

TABLE 7

COVERAGE RATIO (EXP MODEL)
$N = 20, \theta = 2/3, \rho_1 = 1.0, \rho_2 = 10.0$

time C.I (%)	coverage	BIN	NOR	MLE	J.K	B.T
0.5 80 %	too high	21(.07)	50(.17)	33(.11)	140(.47)	222(.74)
	cover	**257(.86)**	228(.76)	233(.78)	**155(.52)**	**77(.26)**
	too low	22(.07)	22(.07)	34(.11)	5(.02)	1(.00)
	int.val	(.82,.90)	(.71,.81)	(.73,.82)	(.46,.57)	(.21,.31)
(.7231) 90 %	too high	3(.01)	21(.07)	18(.06)	103(.34)	187(.62)
	cover	**282(.94)**	264(.88)	268(.89)	**194(.65)**	113(.38)
	too low	15(.05)	15(.05)	14(.05)	3(.01)	0(.00)
	int.val	(.91,.97)	(.84,.92)	(.86,.93)	(.59,.70)	(.32,.43)
1.0 80 %	too high	31(.10)	31(.10)	33(.11)	101(.34)	112(.37)
	cover	231(.77)	231(.77)	233(.78)	**193(.64)**	**176(.59)**
	too low	38(.13)	38(.13)	34(.11)	6(.02)	12(.04)
	int.val	(.72,.82)	(.70,.82)	(.73,.82)	(.59,.70)	(.53,.64)
(.5241) 90 %	too high	14(.05)	32(.11)	18(.06)	69(.23)	76(.25)
	cover	270(.90)	**253(.84)**	268(.89)	**230(.77)**	**217(.72)**
	too low	16(.05)	16(.05)	14(.05)	1(.00)	7(.02)
	int.val	(.87,.93)	(.80,.88)	(.86,.93)	(.72,.81)	(.67,.77)
1.5 80 %	too high	32(.11)	32(.11)	32(.11)	81(.27)	67(.22)
	cover	232(.77)	**199(.66)**	234(.78)	**211(.70)**	**207(.69)**
	too low	36(.12)	69(.23)	34(.11)	8(.03)	26(.09)
	int.val	(.73,.82)	(.61,.72)	(.73,.83)	(.65,.76)	(.64,.74)
(.3799) 90 %	too high	14(.05)	14(.05)	18(.06)	52(.17)	36(.12)
	cover	273(.91)	**250(.83)**	268(.89)	**245(.82)**	**241(.80)**
	too low	13(.04)	36(.12)	14(.05)	3(.01)	23(.08)
	int.val	(.88,.94)	(.79,.88)	(.86,.93)	(.77,.86)	(.76,.85)
2.0 80 %	too high	21(.07)	21(.07)	32(.11)	64(.21)	38(.13)
	cover	251(.84)	**221(.74)**	235(.78)	227(.76)	227(.76)
	too low	28(.09)	58(.19)	33(.11)	9(.03)	35(.12)
	int.val	(.79,.88)	(.69,.79)	(.74,.83)	(.71,.81)	(.71,.81)
(.2753) 90 %	too high	7(.02)	7(.02)	18(.06)	33(.11)	15(.05)
	cover	**287(.96)**	265(.88)	269(.90)	262(.87)	263(.88)
	too low	6(.02)	28(.09)	13(.04)	5(.02)	22(.07)
	int.val	(.93,.98)	(.85,.92)	(.86,.93)	(.83,.91)	(.84,.92)
3.0 80 %	too high	12(.04)	12(.04)	32(.11)	50(.17)	16(.05)
	cover	**274(.91)**	226(.74)	235(.78)	239(.80)	237(.79)
	too low	14(.05)	62(.21)	33(.11)	11(.04)	47(.16)
	int.val	(.88,.95)	(.70,.80)	(.74,.83)	(.75,.84)	(.74,.84)
(.1446) 90 %	too high	4(.01)	4(.01)	18(.06)	28(.09)	9(.03)
	cover	**282(.94)**	234(.78)	270(.90)	267(.89)	263(.88)
	too low	14(.05)	62(.21)	12(.04)	5(.02)	28(.09)
	int.val	(.91,.97)	(.73,.83)	(.87,.93)	(.85,.93)	(.84,.93)
4.0 80 %	too high	21(.07)	21(.07)	32(.11)	44(.15)	15(.05)
	cover	**279(.93)**	**215(.72)**	235(.78)	244(.81)	231(.77)
	too low	0(.00)	64(.21)	33(.11)	12(.04)	54(.18)
	int.val	(.90,.96)	(.67,.77)	(.74,.83)	(.77,.86)	(.71,.82)
(.0760) 90 %	too high	4(.01)	4(.01)	18(.06)	24(.08)	6(.02)
	cover	**296(.98)**	**232(.77)**	270(.90)	271(.90)	261(.87)
	too low	0(.00)	64(.21)	12(.04)	5(.02)	33(.11)
	int.val	(.97,.99)	(.73,.82)	(.87,.93)	(.87,.94)	(.82,.92)

TABLE 8

LENGTH OF C.I FOR ln P{D > t} (EXPONENTIAL MODEL)
$N = 20, \theta = 2/3, \rho_1 = 1.0, \rho_2 = 10.0$

time	C.I (%)	BIN	NOR	MLE	J.K	B.T
0.5	80 %	0.4304 (0.109)	0.3617 (0.104)	0.1981 (0.053)	0.3451 (0.172)	0.3168 (0.128)
	90 %	0.5359 (0.173)	0.4680 (0.139)	0.2544 (0.068)	0.4494 (0.225)	0.4302 (0.183)
1.0	80 %	0.6594 (0.170)	0.5788 (0.161)	0.3952 (0.106)	0.4630 (0.207)	0.4394 (0.164)
	90 %	0.8227 (0.212)	0.7618 (0.225)	0.5075 (0.136)	0.6030 (0.298)	0.5937 (0.242)
1.5	80 %	0.9426 (0.323)	0.8710 (0.469)	0.5931 (0.161)	0.7228 (0.399)	0.6560 (0.256)
	90 %	1.1756 (0.402)	1.2131 (0.773)	0.7616 (0.207)	0.9413 (0.519)	0.8865 (0.379)
2.0	80 %	1.2684 (0.587)	1.2540 (0.899)	0.8300 (0.222)	1.0229 (0.603)	0.9008 (0.359)
	90 %	1.5765 (0.688)	2.0258 (1.817)	0.9311 (0.286)	1.3322 (0.785)	1.2240 (0.517)
3.0	80 %	2.2188 (1.311)	2.5096 (2.079)	1.1951 (0.400)	1.6583 (1.021)	1.4301 (0.574)
	90 %	2.6995 (1.400)	4.1815 (2.807)	1.5347 (0.914)	2.1597 (1.330)	1.9457 (0.830)
4.0	80 %	3.5128 (1.957)	3.1967 (2.724)	1.6119 (0.778)	2.3080 (1.443)	1.9704 (0.796)
	90 %	4.0702 (1.893)	4.8194 (3.183)	2.0699 (0.993)	3.0057 (1.880)	2.6794 (1.140)

C. ROBUSTNESS

The robustness of the estimates in Chapter II was studied with respect to an incorrect model assumption about the distribution of the sojourn time in state 1. In the previous simulations, the maximum likelihood estimator used the known correct model.

The data for the simulation experiment in this section is generated from the following three state semi-Markov process: Individuals start in state 1 at $t = 0$. The probability of a transition to state 0 is θ, and to state 2 is $1 - \theta$. From state 2, the probability of a transition to state 1 is 1. State 0 is an absorbing state. The sojourn

time in state 2 is exponential with mean $1/\rho_2$. The sojourn time in state 1 is the sum of two independent exponentials with means $1/\rho_1$ and $1/\rho_3$; that is, the sojourn time in state 1 has a hypoexponential distribution. The same Fortran program for the simulation is used but slightly modified for the above change. The data generated are analyzed by the same Fortran programs for each estimator as in the first section. In particular, the maximum likelihood estimator assumes the sojourn time in state 1 has an exponential distribution rather than the true hypoexponential distribution.

For the first simulation results reported in Tables 9 to 12, parameter values of $\rho_1 = 2.0$, $\rho_2 = 10.0$, $\rho_3 = 2.0$, and $\theta = 0.5$ are used. Again two different numbers of individuals are used; they are 20 and 50. The simulation is replicated 300 times, the coverage numbers and the average lengths of the confidence intervals are computed. The actual value of the survival function is computed by inverting the Laplace transform of the passage time to state 0 for the semi-Markov process. Some computed values of the survivor function can be found in Appendix B.

Table 9 reports coverage results for the case $N = 50$ individuals. The use of the incorrect model for the maximum likelihood estimator leads to the majority of the intervals being too low for $t=0.5$ and $t=1.0$. The binomial confidence interval has a slight tendency to overcover. The confidence interval based on the asymptotic renewal estimator undercover for $t=0.5$ and $t=1.0$. For larger values of t they have the correct coverage. The results for the jackknife and bootstrap individuals are similar to those in Table 1. Table 10 reports the average lengths of the confidence intervals. The average lengths for the maximum likelihood procedure are similar to those of Table 2. For the other procedures, the average lengths are smaller.

Results of the simulation experiment for $N = 20$ individuals are given in Table 11 and 12. Table 11 shows the coverage results. The maximum likelihood procedure undercovers for $t=0.5$ and $t=1.0$ and overcovers for $t=1.5$ and $t=2.0$. The binomial procedure once again tends to overcover. The procedures based on the asymptotic renewal estimator have the correct coverage for all t except $t=0.5$. The average lengths of the intervals are given in Table 12.

Tables 13 to 16 report results of simulation of a semi-Markov model with $\theta = 2/3$ but with the other parameters the same; $\rho_1 = 2.0$, $\rho_2 = 10.0$, $\rho_3 = 2.0$.

Results for $N = 20$ individuals appear in Tables 15 and 16. The confidence intervals based on the asymptotic renewal estimator have the correct coverage for all t except 0.5 and 1.0. The results are similar to those of Tables 11 and 12.

TABLE 9

COVERAGE RATIO (HYPOEXP MODEL)
$N = 50, \theta = 0.5, \rho_1 = 2.0, \rho_2 = 10.0, \rho_3 = 2.0$

time C.I (%)	length	BIN	NOR	MLE	J.K	B.T
0.5 80 %	too high	18(.06)	35(.12)	0(.00)	130(.43)	149(.50)
	cover	**261(.87)**	244(.81)	**3(.01)**	**165(.55)**	**150(.50)**
	too low	21(.07)	21(.07)	297(.99)	5(.02)	1(.00)
	int.val	(.83,.91)	(.77,.86)	(.00,.01)	(.49,.61)	(.44,.56)
(.8652) 90 %	too high	10(.03)	18(.06)	0(.00)	99(.33)	106(.35)
	cover	**283(.94)**	275(.92)	**6(.02)**	**200(.67)**	**194(.65)**
	too low	7(.02)	7(.02)	294(.98)	1(.00)	0(.00)
	int.val	(.92,.97)	(.89,.95)	(.00,.04)	(.61,.72)	(.59,.70)
1.0 80 %	too high	22(.07)	3(.01)	2(.01)	42(.14)	39(.13)
	cover	**260(.87)**	229(.76)	**145(.48)**	242(.81)	240(.80)
	too low	18(.06)	33(.11)	153(.51)	16(.05)	21(.07)
	int.val	(.83,.91)	(.72,.81)	(.43,.54)	(.76,.85)	(.75,.85)
(.6743) 90 %	too high	11(.04)	22(.07)	0(.00)	28(.09)	21(.07)
	cover	280(.93)	269(.90)	**213(.71)**	263(.88)	265(.88)
	too low	9(.03)	9(.03)	87(.29)	9(.03)	14(.05)
	int.val	(.91,.96)	(.86,.93)	(.66,.76)	(.84,.91)	(.85,.92)
1.5 80 %	too high	27(.09)	27(.09)	6(.02)	31(.10)	25(.08)
	cover	254(.85)	235(.78)	242(.81)	247(.82)	243(.81)
	too low	19(.06)	38(.13)	52(.17)	22(.07)	32(.11)
	int.val	(.81,.89)	(.74,.83)	(.76,.85)	(.78,.87)	(.77,.85)
(.5157) 90 %	too high	11(.04)	21(.07)	4(.01)	21(.07)	19(.06)
	cover	276(.92)	261(.87)	276(.93)	271(.90)	262(.87)
	too low	13(.04)	18(.06)	20(.07)	8(.02)	19(.06)
	int.val	(.89,.95)	(.83,.91)	(.89,.95)	(.87,.94)	(.84,.91)
2.0 80 %	too high	25(.08)	25(.08)	13(.04)	29(.10)	24(.08)
	cover	253(.84)	240(.80)	**264(.88)**	250(.83)	239(.80)
	too low	22(.07)	35(.12)	23(.08)	21(.07)	37(.12)
	int.val	(.80,.88)	(.75,.85)	(.84,.92)	(.79,.88)	(.75,.84)
(.3930) 90 %	too high	16(.05)	16(.05)	7(.02)	20(.07)	12(.04)
	cover	274(.91)	262(.87)	**282(.94)**	269(.90)	264(.88)
	too low	10(.03)	22(.07)	11(.04)	11(.04)	24(.08)
	int.val	(.88,.95)	(.84,.91)	(.91,.97)	(.86,.93)	(.84,.92)
3.0 80 %	too high	25(.08)	25(.08)	34(.11)	31(.10)	21(.07)
	cover	247(.82)	**220(.73)**	253(.84)	248(.83)	241(.80)
	too low	28(.09)	55(.18)	13(.04)	21(.07)	38(.13)
	int.val	(.78,.87)	(.68,.78)	(.80,.88)	(.78,.87)	(.76,.85)
(.2278) 90 %	too high	17(.06)	17(.06)	20(.07)	18(.06)	11(.04)
	cover	273(.91)	**255(.85)**	275(.92)	271(.90)	266(.89)
	too low	10(.03)	28(.09)	5(.02)	11(.04)	23(.08)
	int.val	(.88,.94)	(.81,.89)	(.89,.95)	(.87,.94)	(.85,.92)
4.0 80 %	too high	11(.04)	11(.04)	47(.16)	31(.10)	20(.07)
	cover	**266(.89)**	235(.78)	246(.82)	244(.81)	238(.79)
	too low	23(.08)	54(.18)	7(.02)	25(.08)	42(.14)
	int.val	(.85,.92)	(.74,.83)	(.78,.86)	(.77,.86)	(.75,.84)
(.1320) 90 %	too high	6(.02)	6(.02)	29(.10)	19(.06)	10(.03)
	cover	**285(.95)**	271(.90)	267(.89)	270(.90)	264(.88)
	too low	9(.03)	23(.08)	4(.01)	11(.04)	26(.09)
	int.val	(.93,.97)	(.87,.94)	(.85,.93)	(.87,.93)	(.84,.92)

TABLE 10

LENGTH OF C.I FOR ln P{D > t}(HYPOEXP MODEL)
$N = 50, \theta = 0.5, \rho_1 = 2.0, \rho_2 = 10.0, \rho_3 = 2.0$

time	C.I (%)	BIN	NOR	MLE	J.K	B.T
0.5	80 %	0.1688 (0.029)	0.1442 (0.029)	0.0886 (0.011)	0.0913 (0.042)	0.0806 (0.022)
	90 %	0.2110 (0.037)	0.1852 (0.037)	0.1138 (0.014)	0.1210 (0.056)	0.1043 (0.028)
1.0	80 %	0.2843 (0.043)	0.2553 (0.041)	0.1764 (0.022)	0.1666 (0.053)	0.1534 (0.028)
	90 %	0.3572 (0.054)	0.3291 (0.053)	0.2265 (0.028)	0.2208 (0.071)	0.1988 (0.037)
1.5	80 %	0.3958 (0.058)	0.3606 (0.054)	0.2641 (0.033)	0.2692 (0.083)	0.2496 (0.047)
	90 %	0.4978 (0.073)	0.4666 (0.072)	0.3392 (0.042)	0.3568 (0.110)	0.3240 (0.064)
2.0	80 %	0.5139 (0.080)	0.4710 (0.075)	0.3519 (0.044)	0.3777 (0.117)	0.3506 (0.067)
	90 %	0.6461 (0.100)	0.6130 (0.100)	0.4519 (0.056)	0.5006 (0.155)	0.4561 (0.093)
3.0	80 %	0.7840 (0.160)	0.7261 (0.157)	0.5274 (0.066)	0.5998 (0.188)	0.5576 (0.111)
	90 %	0.9832 (0.199)	0.9684 (0.247)	0.6773 (0.085)	0.7950 (0.249)	0.7261 (0.153)
4.0	80 %	1.1592 (0.334)	1.1276 (0.480)	0.7030 (1.088)	0.8242 (0.260)	0.7650 (0.154)
	90 %	1.4496 (0.415)	1.6667 (1.017)	0.9027 (1.113)	1.0924 (0.345)	0.9980 (0.214)

TABLE 11

COVERAGE RATIO (HYPOEXP MODEL)
$N = 20,\ \theta = 0.5,\ \rho_1 = 2.0,\ \rho_2 = 10.0,\ \rho_3 = 2.0$

time	C.I (%)	length	BIN	NOR	MLE	J.K	B.T
0.5	80 %	too high	12(.04)	59(.20)	0(.00)	105(.35)	135(.45)
		cover	**278(.93)**	231(.77)	**53(.18)**	**183(.61)**	**162(.54)**
		too low	10(.03)	10(.03)	247(.82)	12(.04)	3(.01)
		int.val	(.90,.96)	(.72,.82)	(.13,.22)	(.55,.67)	(.48,.60)
(.8652)	90 %	too high	0(.00)	37(.12)	0(.00)	73(.24)	82(.37)
		cover	**290(.97)**	260(.87)	**109(.36)**	**220(.73)**	**217(.72)**
		too low	10(.03)	3(.01)	191(.64)	7(.02)	1(.00)
		int.val	(.95,.99)	(.83,.92)	(.31,.42)	(.68,.78)	(.67,.77)
1.0	80 %	too high	16(.05)	44(.15)	1(.00)	45(.15)	32(.11)
		cover	254(.85)	226(.75)	**221(.74)**	240(.80)	241(.80)
		too low	30(.10)	30(.10)	78(.26)	15(.05)	27(.09)
		int.val	(.80,.88)	(.70,.80)	(.69,.79)	(.75,.85)	(.76,.85)
(.6743)	90 %	too high	5(.02)	16(.05)	0(.00)	29(.10)	11(.04)
		cover	**285(.95)**	274(.91)	266(.89)	263(.88)	271(.90)
		too low	10(.03)	10(.03)	34(.11)	8(.02)	18(.06)
		int.val	(.93,.97)	(.88,.95)	(.85,.92)	(.84,.91)	(.87,.94)
1.5	80 %	too high	21(.07)	21(.07)	8(.03)	40(.13)	23(.08)
		cover	**260(.87)**	242(.81)	**257(.86)**	244(.81)	232(.77)
		too low	19(.06)	37(.12)	35(.12)	16(.05)	45(.15)
		int.val	(.83,.91)	(.76,.85)	(.82,.90)	(.77,.86)	(.73,.82)
(.5157)	90 %	too high	5(.02)	21(.07)	2(.01)	24(.08)	7(.02)
		cover	276(.92)	260(.87)	**283(.94)**	273(.91)	264(.88)
		too low	19(.06)	19(.06)	15(.05)	3(.01)	29(.10)
		int.val	(.89,.95)	(.83,.91)	(.92,.97)	(.88,.94)	(.84,.92)
2.0	80 %	too high	11(.04)	28(.09)	17(.06)	39(.13)	18(.06)
		cover	**270(.90)**	235(.78)	**260(.87)**	246(.82)	232(.77)
		too low	19(.06)	37(.12)	23(.08)	15(.05)	50(.17)
		int.val	(.87,.93)	(.74,.83)	(.83,.91)	(.78,.86)	(.73,.82)
(.3930)	90 %	too high	11(.04)	11(.04)	7(.02)	19(.06)	7(.02)
		cover	**284(.95)**	270(.90)	**284(.95)**	277(.92)	263(.88)
		too low	5(.02)	19(.06)	9(.03)	4(.01)	30(.10)
		int.val	(.92,.97)	(.87,.93)	(.92,.97)	(.89,.95)	(.84,.91)
3.0	80 %	too high	13(.04)	13(.04)	31(.10)	37(.12)	17(.06)
		cover	**273(.91)**	249(.83)	254(.85)	251(.84)	230(.77)
		too low	14(.05)	38(.13)	15(.05)	12(.04)	53(.18)
		int.val	(.88,.94)	(.79,.87)	(.81,.89)	(.79,.88)	(.72,.81)
(.2278)	90 %	too high	5(.02)	5(.02)	17(.06)	19(.06)	6(.02)
		cover	**281(.94)**	**257(.86)**	279(.93)	277(.92)	261(.87)
		too low	14(.05)	38(.13)	4(.01)	4(.01)	33(.11)
		int.val	(.91,.96)	(.82,.90)	(.90,.96)	(.89,.95)	(.83,.91)
4.0	80 %	too high	4(.01)	4(.01)	44(.15)	38(.13)	17(.06)
		cover	**279(.93)**	226(.75)	245(.82)	251(.84)	229(.76)
		too low	17(.06)	70(.23)	11(.04)	11(.04)	54(.18)
		int.val	(.87,.94)	(.70,.80)	(.79,.88)	(.81,.89)	(.72,.82)
(.1320)	90 %	too high	4(.01)	2(.01)	23(.08)	19(.06)	6(.02)
		cover	**296(.98)**	**228(.76)**	273(.91)	277(.92)	260(.87)
		too low	0(.00)	70(.23)	4(.01)	4(.01)	34(.11)
		int.val	(.97,.99)	(.71,.81)	(.88,.94)	(.89,.95)	(.83,.91)

TABLE 12

LENGTH OF C.I FOR ln P{D > t}(HYPOEXP MODEL)
$N = 20, \theta = 0.5, \rho_1 = 2.0, \rho_2 = 10.0, \rho_3 = 2.0$

time	C.I (%)	BIN	NOR	MLE	J.K	B.T
0.5	80 %	0.2891 (0.077)	0.2217 (0.083)	0.1438 (0.030)	0.1405 (0.056)	0.1296 (0.046)
	90 %	0.3591 (0.095)	0.2810 (0.111)	0.1846 (0.038)	0.1829 (0.075)	0.1750 (0.066)
1.0	80 %	0.4845 (0.113)	0.4131 (0.107)	0.2861 (0.059)	0.2694 (0.105)	0.2527 (0.073)
	90 %	0.6038 (0.142)	0.5365 (0.143)	0.3674 (0.076)	0.3509 (0.136)	0.3314 (0.102)
1.5	80 %	0.6810 (0.175)	0.5995 (0.168)	0.4284 (0.088)	0.4512 (0.180)	0.4190 (0.128)
	90 %	0.8496 (0.218)	0.7911 (0.241)	0.5501 (0.113)	0.5876 (0.234)	0.5517 (0.174)
2.0	80 %	0.8843 (0.230)	0.7974 (0.240)	0.5707 (0.118)	0.6432 (0.259)	0.5944 (0.290)
	90 %	1.1030 (0.286)	1.0914 (0.513)	0.7329 (0.151)	0.8376 (0.337)	0.7864 (0.256)
3.0	80 %	1.4552 (0.589)	1.5826 (1.296)	0.8553 (0.176)	1.0345 (0.419)	0.9528 (0.313)
	90 %	1.8145 (0.737)	2.4881 (1.984)	1.0983 (0.227)	1.3472 (0.546)	1.2605 (0.427)
4.0	80 %	2.3431 (1.375)	2.6289 (2.141)	1.1399 (0.235)	1.4289 (0.581)	1.3138 (0.437)
	90 %	2.8401 (1.447)	4.3272 (2.778)	1.4638 (0.302)	1.8610 (0.757)	1.7390 (0.601)

TABLE 13

COVERAGE RATIO (HYPOEXP MODEL)

$N = 50, \theta = 2/3, \rho_1 = 2.0, \rho_2 = 10.0, \rho_3 = 2.0$

time	C.I (%)	length	BIN	NOR	MLE	J.K	B.T
0.5	80 %	too high	25(.08)	25(.08)	0(.00)	186(.62)	220(.73)
		cover	249(.83)	249(.83)	**2(.01)**	**109(.36)**	**80(.27)**
		too low	26(.09)	26(.09)	298(.99)	5(.02)	0(.00)
		int.val	(.78,.87)	(.78,.87)	(.00,.02)	(.31,.42)	(.22,.32)
(.8214)	90 %	too high	8(.03)	25(.08)	0(.00)	157(.52)	194(.65)
		cover	**284(.95)**	267(.89)	**4(.01)**	**141(.47)**	**106(.35)**
		too low	8(.03)	8(.03)	296(.99)	2(.01)	0(.00)
		int.val	(.92,.97)	(.85,.93)	(.00,.03)	(.41,.53)	(.30,.41)
1.0	80 %	too high	30(.10)	30(.10)	0(.00)	65(.22)	59(.20)
		cover	254(.85)	244(.81)	**179(.60)**	**219(.73)**	**225(.76)**
		too low	16(.05)	26(.09)	121(.40)	11(.04)	16(.05)
		int.val	(.81,.89)	(.77,.86)	(.54,.65)	(.68,.78)	(.71,.81)
(.5788)	90 %	too high	10(.03)	15(.05)	0(.00)	39(.13)	32(.11)
		cover	280(.93)	269(.90)	**237(.79)**	**252(.84)**	261(.87)
		too low	10(.03)	16(.05)	63(.21)	9(.03)	7(.02)
		int.val	(.91,.97)	(.86,.93)	(.74,.84)	(.80,.88)	(.83,.91)
1.5	80 %	too high	19(.06)	19(.06)	8(.03)	42(.14)	29(.10)
		cover	253(.84)	242(.81)	**264(.88)**	241(.80)	240(.80)
		too low	28(.09)	39(.13)	28(.09)	17(.06)	31(.10)
		int.val	(.80,.88)	(.76,.85)	(.84,.92)	(.76,.85)	(.75,.85)
(.3936)	90 %	too high	11(.04)	11(.04)	4(.01)	17(.06)	15(.05)
		cover	274(.91)	261(.87)	**284(.95)**	275(.92)	265(.88)
		too low	15(.05)	28(.09)	12(.04)	8(.03)	20(.07)
		int.val	(.88,.95)	(.83,.91)	(.92,.97)	(.89,.95)	(.85,.92)
2.0	80 %	too high	26(.09)	26(.09)	27(.09)	35(.12)	24(.08)
		cover	249(.83)	233(.78)	**261(.87)**	249(.83)	235(.78)
		too low	25(.08)	41(.14)	12(.04)	16(.07)	41(.14)
		int.val	(.79,.87)	(.73,.82)	(.83,.91)	(.79,.87)	(.74,.83)
(.2652)	90 %	too high	5(.02)	14(.05)	11(.04)	18(.06)	12(.04)
		cover	**283(.94)**	261(.87)	**285(.95)**	274(.91)	262(.87)
		too low	12(.04)	25(.08)	4(.01)	8(.03)	26(.09)
		int.val	(.92,.97)	(.83,.91)	(.93,.97)	(.88,.95)	(.84,.91)
3.0	80 %	too high	14(.05)	14(.05)	69(.23)	35(.12)	22(.07)
		cover	**271(.90)**	243(.81)	227(.76)	249(.83)	229(.76)
		too low	15(.05)	43(.14)	4(.01)	16(.05)	49(.16)
		int.val	(.87,.94)	(.77,.85)	(.71,.81)	(.79,.87)	(.72,.81)
(.1197)	90 %	too high	5(.02)	5(.02)	37(.12)	17(.06)	6(.02)
		cover	**292(.97)**	**252(.84)**	261(.87)	275(.92)	261(.87)
		too low	3(.01)	43(.14)	2(.01)	8(.03)	33(.11)
		int.val	(.96,.99)	(.80,.88)	(.83,.91)	(.86,.95)	(.83,.91)
4.0	80 %	too high	13(.04)	13(.04)	98(.33)	34(.11)	19(.06)
		cover	**275(.92)**	**221(.74)**	**199(.66)**	248(.83)	229(.76)
		too low	12(.04)	66(.22)	3(.01)	18(.06)	52(.17)
		int.val	(.89,.95)	(.69,.79)	(.61,.72)	(.78,.87)	(.72,.81)
(.0540)	90 %	too high	4(.01)	4(.01)	63(.21)	15(.05)	6(.02)
		cover	296(.98)	230(.77)	**237(.79)**	277(.92)	260(.87)
		too low	0(.00)	66(.22)	0(.00)	8(.03)	34(.11)
		int.val	(.97,.99)	(.72,.81)	(.74,.84)	(.89,.95)	(.83,.91)

TABLE 14

LENGTH OF C.I FOR ln P{D > t}(HYPOEXP MODEL)
$N = 50, \theta = 2/3, \rho_1 = 2.0, \rho_2 = 10.0, \rho_3 = 2.0$

time	C.I (%)	BIN	NOR	MLE	J.K	B.T
0.5	80 %	0.1956 (0.032)	0.1704 (0.031)	0.1194 (0.014)	0.1496 (0.057)	0.1351 (0.035)
	90 %	0.2450 (0.040)	0.2197 (0.040)	0.1533 (0.018)	0.1983 (0.076)	0.1773 (0.046)
1.0	80 %	0.3463 (0.049)	0.3140 (0.046)	0.2378 (0.028)	0.2267 (0.069)	0.2140 (0.042)
	90 %	0.4355 (0.062)	0.4056 (0.060)	0.3054 (0.036)	0.3005 (0.091)	0.2768 (0.053)
1.5	80 %	0.5128 (0.080)	0.4700 (0.075)	0.3563 (0.042)	0.3751 (0.012)	0.3537 (0.072)
	90 %	0.6447 (0.100)	0.6117 (0.100)	0.4576 (0.054)	0.4972 (0.159)	0.4588 (0.095)
2.0	80 %	0.7076 (0.143)	0.6531 (0.135)	0.4748 (0.057)	0.5405 (0.174)	0.4977 (0.106)
	90 %	0.8881 (0.176)	0.8630 (0.193)	0.6097 (0.073)	0.7164 (0.232)	0.6584 (0.144)
3.0	80 %	1.2845 (0.532)	1.2267 (0.526)	0.7118 (0.085)	0.8833 (0.287)	0.8240 (0.182)
	90 %	1.5970 (0.591)	1.8844 (1.190)	0.9141 (0.109)	1.1708 (0.380)	1.0761 (0.240)
4.0	80 %	2.2724 (1.102)	2.6235 (1.778)	0.9488 (0.113)	1.2309 (0.402)	1.1452 (0.259)
	90 %	2.7889 (1.216)	4.1527 (2.227)	1.2184 (0.145)	1.6314 (0.532)	1.4950 (0.336)

TABLE 15

COVERAGE RATIO (HYPOEXP MODEL)
$N = 20, \theta = 2/3, \rho_1 = 2.0, \rho_2 = 10.0, \rho_3 = 2.0$

time	C.I (%)	length	BIN	NOR	MLE	J.K	B.T
0.5	80 %	too high	3(.01)	26(.09)	0(.00)	124(.41)	198(.66)
		cover	**277(.92)**	254(.85)	**54(.18)**	**172(.57)**	**101(.34)**
		too low	20(.07)	20(.07)	246(.82)	4(.01)	1(.00)
		int.val	(.89,.95)	(.81,.88)	(.14,.22)	(.52,.63)	(.28,.39)
(.8214)	90 %	too high	3(.01)	26(.09)	0(.00)	88(.29)	144(.48)
		cover	**288(.96)**	265(.88)	**115(.38)**	**211(.70)**	**156(.52)**
		too low	9(.03)	9(.03)	185(.62)	1(.00)	0(.00)
		int.val	(.94,.98)	(.85,.92)	(.33,.44)	(.65,.76)	(.28,.39)
1.0	80 %	too high	25(.08)	25(.08)	0(.00)	62(.21)	57(.19)
		cover	242(.81)	242(.81)	238(.79)	**222(.74)**	**216(.72)**
		too low	33(.11)	33(.11)	62(.21)	16(.05)	27(.09)
		int.val	(.76,.85)	(.76,.85)	(.75,.84)	(.69,.79)	(.67,.77)
(.5788)	90 %	too high	13(.04)	22(.07)	0(.00)	37(.12)	20(.07)
		cover	267(.89)	260(.87)	274(.91)	**253(.84)**	261(.87)
		too low	20(.07)	18(.06)	26(.09)	10(.03)	19(.06)
		int.val	(.85,.93)	(.83,.91)	(.88,.95)	(.80,.88)	(.83,.91)
1.5	80 %	too high	16(.05)	34(.11)	11(.04)	49(.16)	25(.08)
		cover	**257(.86)**	**218(.73)**	**264(.88)**	237(.79)	231(.77)
		too low	27(.09)	48(.16)	25(.08)	14(.05)	44(.15)
		int.val	(.82,.90)	(.68,.78)	(.84,.92)	(.74,.84)	(.72,.82)
(.3936)	90 %	too high	7(.02)	16(.06)	4(.01)	22(.07)	8(.03)
		cover	**283(.94)**	**257(.86)**	**284(.95)**	269(.90)	264(.88)
		too low	10(.03)	27(.09)	12(.04)	9(.03)	28(.09)
		int.val	(.92,.97)	(.82,.90)	(.92,.97)	(.86,.93)	(.84,.92)
2.0	80 %	too high	16(.05)	16(.05)	24(.08)	41(.13)	23(.08)
		cover	245(.82)	**225(.75)**	**259(.86)**	243(.81)	231(.77)
		too low	39(.13)	59(.20)	17(.06)	16(.07)	46(.15)
		int.val	(.77,.86)	(.70,.80)	(.82,.90)	(.77,.85)	(.72,.82)
(.2652)	90 %	too high	5(.02)	16(.05)	11(.04)	20(.07)	7(.02)
		cover	**283(.94)**	**245(.82)**	**281(.94)**	271(.90)	263(.88)
		too low	12(.04)	39(.13)	8(.03)	9(.03)	30(.10)
		int.val	(.92,.97)	(.77,.86)	(.91,.96)	(.87,.94)	(.83,.92)
3.0	80 %	too high	20(.07)	20(.07)	58(.19)	39(.13)	18(.06)
		cover	**257(.86)**	**189(.63)**	231(.77)	242(.81)	226(.75)
		too low	23(.08)	91(.30)	11(.04)	19(.06)	56(.19)
		int.val	(.82,.90)	(.58,.68)	(.72,.82)	(.76,.85)	(.70,.80)
(.1197)	90 %	too high	3(.01)	3(.01)	26(.09)	19(.06)	6(.02)
		cover	**297(.99)**	274(.91)	269(.90)	275(.92)	262(.87)
		too low	0(.00)	23(.08)	5(.02)	6(.02)	32(.11)
		int.val	(.98,.99)	(.88,.95)	(.86,.93)	(.89,.95)	(.83,.92)
4.0	80 %	too high	26(.09)	6(.02)	65(.22)	36(.12)	15(.05)
		cover	**274(.91)**	**196(.65)**	226(.75)	247(.82)	229(.76)
		too low	0(.00)	98(.33)	9(.03)	17(.06)	56(.19)
		int.val	(.88,.95)	(.60,.71)	(.71,.80)	(.78,.87)	(.72,.81)
(.0540)	90 %	too high	6(.02)	0(.00)	41(.14)	16(.05)	6(.02)
		cover	294(.98)	**202(.67)**	**256(.85)**	278(.93)	260(.87)
		too low	0(.00)	98(.33)	3(.01)	6(.02)	34(.11)
		int.val	(.96,.99)	(.62,.73)	(.81,.89)	(.90,.96)	(.83,.91)

TABLE 16

LENGTH OF C.I FOR ln P{D > t}(HYPOEXP MODEL)
$N = 20, \theta = 2/3, \rho_1 = 2.0, \rho_2 = 10.0, \rho_3 = 2.0$

time	C.I (%)	BIN	NOR	MLE	J.K	B.T
0.5	80 %	0.3368 (0.088)	0.2706 (0.088)	0.1951 (0.042)	0.2539 (0.149)	0.2382 (0.105)
	90 %	0.4186 (0.110)	0.3466 (0.119)	0.2505 (0.054)	0.3306 (0.194)	0.3249 (0.143)
1.0	80 %	0.6008 (0.158)	0.5234 (0.150)	0.3892 (0.084)	0.3918 (0.161)	0.3675 (0.124)
	90 %	0.7494 (0.197)	0.6858 (0.208)	0.4998 (0.108)	0.5103 (0.209)	0.4872 (0.174)
1.5	80 %	0.9084 (0.310)	0.8374 (0.465)	0.5840 (0.129)	0.6753 (0.305)	0.6142 (0.213)
	90 %	1.1331 (0.387)	1.1701 (0.834)	0.7500 (0.166)	0.8794 (0.397)	0.8144 (0.298)
2.0	80 %	1.3743 (0.845)	1.3323 (1.078)	0.7800 (0.183)	0.9871 (0.470)	0.8909 (0.316)
	90 %	1.6939 (0.930)	2.1678 (2.013)	1.0016 (0.235)	1.2855 (0.612)	1.1786 (0.443)
3.0	80 %	2.5888 (1.497)	2.9300 (2.330)	1.1776 (0.357)	1.6313 (0.810)	1.4596 (0.534)
	90 %	3.1180 (1.550)	4.7553 (2.863)	1.5122 (0.462)	2.1245 (1.055)	1.9376 (0.737)
4.0	80 %	4.2251 (2.031)	3.3399 (2.307)	1.5894 (0.763)	2.2834 (1.154)	2.0356 (0.749)
	90 %	4.7925 (1.883)	4.5249 (3.430)	2.0410 (0.980)	2.9739 (1.503)	2.7125 (1.038)

IV. CONCLUSIONS

This thesis considers the problem of estimating the log probability for a semi-Markov process which does not enter a particular state before time t.

Simulation is used to study procedures for obtaining confidence intervals for the ln probability a semi-Markov process enters a fixed state after time t. The data arise from observing N individuals. Three estimators and associated confidence interval procedures are compared: The three estimators use different amounts of information about the process. The maximum likelihood estimator and its normal confidence interval uses the most information. An estimator based on the observed first passage times uses the least. An estimator based on an exponential approximation to the survivor function of the first passage time uses an intermediate amount of information: confidence intervals for this last estimator are obtained using jackknife and bootstrap procedures. The simulation results indicate the following.

(1) Larger numbers of individuals result in smaller average confidence interval lengths.

(2) The binomial confidence interval tends to overcover. This is true in general since the target coverage probability is a lower bound on the true coverage probability. They also tend to have larger average length.

(3) The maximum likelihood confidence intervals have the correct coverage if the model on which they are based is correct. If the model is incorrect they can either overcover or undercover.

(4) If a jackknife interval does not cover the true value the interval will tend to be 'too high'.

(5) The confidence intervals using the asymptotic renewal estimator have the correct coverage for largish t.

(6) The bootstrap confidence interval requires much more computation than the jackknife confidence interval. Since the results for the jackknife and bootstrap procedures are similiar, ease of computation appears to favor the jackknife procedure.

APPENDIX A
TRUE PROBABILITY TABLE I

TABLE 17
EXPONENTIAL MODEL P{D > TIME}.

time	$\rho_1 = 1.0$,$\rho_2 = 10.0$ $\theta = 0.5$	$\rho_1 = 1.0$,$\rho_2 = 2.0$ $\theta = 0.5$	$\rho_1 = 1.0$,$\rho_2 = 10.0$ $\theta = 0.6667$
0.0	1.0000	1.0000	1.0000
0.5	0.7866	0.7968	0.7231
1.0	0.6203	0.6503	0.5241
1.5	0.4891	0.5351	0.3799
2.0	0.3857	0.4415	0.2753
2.5	0.3042	0.3646	0.1995
3.0	0.2399	0.3012	0.1446
3.5	0.1891	0.2488	0.1048
4.0	0.1492	0.2055	0.0760
4.5	0.1176	0.1698	0.0551
5.0	0.0928	0.1403	0.0399
5.5	0.0731	0.1159	0.0289
6.0	0.0577	0.0957	0.0210
6.5	0.0455	0.0791	0.0152
7.0	0.0359	0.0653	0.0110
7.5	0.0283	0.0540	0.0080
8.0	0.0223	0.0446	0.0058
8.5	0.0176	0.0368	0.0042
9.0	0.0139	0.0304	0.0030

The sojourn time in state 1 has an exponential distribution with mean $1/\rho_1$. The sojourn time in state 2 has an exponential distribution with mean $1/\rho_2$.

APPENDIX B
TRUE PROBABILITY TABLE II

TABLE 18

HYPOEXPONENTIAL MODEL P{D > TIME}.

time	$\rho_1 = 2.0$, $\rho_2 = 10.0$ $\rho_3 = 2.0$, $\theta = 0.5$	$\rho_1 = 2.0$, $\rho_2 = 2.0$ $\rho_3 = 2.0$, $\theta = 0.5$	$\rho_1 = 2.0$, $\rho_2 = 10.0$ $\rho_3 = 2.0$, $\theta = 0.6667$
0.0	1.0000	1.0000	1.0000
0.5	0.8652	0.8670	0.8214
1.0	0.6743	0.6897	0.5788
1.5	0.5157	0.5519	0.3936
2.0	0.3930	0.4465	0.2652
2.5	0.2992	0.3268	0.1783
3.0	0.2278	0.2952	0.1197
3.5	0.1734	0.2402	0.0804
4.0	0.1320	0.1954	0.0540
4.5	0.1005	0.1590	0.0363
5.0	0.0765	0.1293	0.0244
5.5	0.0582	0.1052	0.0164
6.0	0.0443	0.0856	0.0110
6.5	0.0337	0.0697	0.0074
7.0	0.0257	0.0567	0.0050
7.5	0.0196	0.0461	0.0033
8.0	0.0149	0.0375	0.0022
8.5	0.0113	0.0305	0.0015
9.0	0.0086	0.0248	0.0010

The sojourn time in state 1 is the sum of exponential distribution with mean $1/\rho_1$ and mean $1/\rho_3$. The sojourn time in state 2 has an exponential distribution with mean $1/\rho_2$.

APPENDIX C
AVERAGE LENGTH OF C.I FOR P{D > T}

Recorded in the Appendix are the average length of confidence intervals for P{D > t} corresponding to those given in the thesis for ln P{D > t}. If a confidence limit is greater than 1, it is set equal to 1. If a confidence limit is less than 0, it is set equal to 0. The results are similar to those in the previous tables.

TABLE 18

LENGTH OF C.I FOR P{D > t} (EXPONENTIAL MODEL)
$N = 50$, $\theta = 0.5$, $\rho_1 = 1.0$, $\rho_2 = 10.0$

time	C.I (%)	BIN	NOR	MLE	J.K	$\bar{B}$.T
0.5	80 %	0.1651 (0.014)	0.1469 (0.014)	0.0692 (0.007)	0.1259 (0.058)	0.1044 (0.027)
	90 %	0.2054 (0.018)	0.1887 (0.019)	0.0889 (0.009)	0.1669 (0.077)	0.1330 (0.033)
1.0	80 %	0.1915 (0.006)	0.1741 (0.007)	0.1078 (0.007)	0.1278 (0.036)	0.1154 (0.015)
	90 %	0.2389 (0.008)	0.2236 (0.009)	0.1384 (0.009)	0.1694 (0.048)	0.1483 (0.018)
1.5	80 %	0.1967 (0.002)	0.1795 (0.003)	0.1264 (0.005)	0.1381 (0.035)	0.1289 (0.014)
	90 %	0.2455 (0.003)	0.2305 (0.003)	0.1623 (0.006)	0.1830 (0.047)	0.1650 (0.019)
2.0	80 %	0.1913 (0.007)	0.1740 (0.007)	0.1320 (0.002)	0.1427 (0.035)	0.1334 (0.015)
	90 %	0.2387 (0.008)	0.2234 (0.009)	0.1695 (0.002)	0.1892 (0.046)	0.1711 (0.019)
3.0	80 %	0.1706 (0.012)	0.1526 (0.013)	0.1220 (0.008)	0.1325 (0.031)	0.1233 (0.016)
	90 %	0.2123 (0.016)	0.1960 (0.016)	0.1567 (0.010)	0.1756 (0.041)	0.1569 (0.021)
4.0	80 %	0.1459 (0.017)	0.1271 (0.017)	0.1007 (0.013)	0.1092 (0.028)	0.1009 (0.018)
	90 %	0.1820 (0.021)	0.1631 (0.023)	0.1293 (0.017)	0.1447 (0.037)	0.1283 (0.023)

TABLE 19

LENGTH OF C.I FOR $P\{D > t\}$ (EXPONENTIAL MODEL)
$N = 20$, $\theta = 0.5$, $\rho_1 = 1.0$, $\rho_2 = 10.0$

time	C.I (%)	BIN	NOR	MLE	J.K	B.T
0.5	80 %	0.2689 (0.037)	0.2262 (0.045)	0.1111 (0.018)	0.1746 (0.065)	0.1584 (0.055)
	90 %	0.3281 (0.046)	0.2891 (0.060)	0.1426 (0.023)	0.2274 (0.085)	0.2094 (0.078)
1.0	80 %	0.3087 (0.020)	0.2702 (0.023)	0.1708 (0.018)	0.1913 (0.050)	0.1820 (0.035)
	90 %	0.3775 (0.024)	0.3469 (0.030)	0.2194 (0.023)	0.2492 (0.065)	0.2356 (0.047)
1.5	80 %	0.3169 (0.010)	0.2792 (0.011)	0.1985 (0.010)	0.2165 (0.051)	0.2030 (0.029)
	90 %	0.3876 (0.013)	0.3585 (0.014)	0.2549 (0.013)	0.2820 (0.067)	0.2617 (0.040)
2.0	80 %	0.3082 (0.019)	0.2698 (0.021)	0.2058 (0.007)	0.2244 (0.049)	0.2077 (0.028)
	90 %	0.3768 (0.024)	0.3463 (0.027)	0.2643 (0.009)	0.2923 (0.064)	0.2650 (0.037)
3.0	80 %	0.2758 (0.032)	0.2345 (0.037)	0.1886 (0.022)	0.2027 (0.049)	0.1854 (0.034)
	90 %	0.3365 (0.040)	0.3001 (0.049)	0.2422 (0.029)	0.2640 (0.063)	0.2338 (0.044)
4.0	80 %	0.2377 (0.045)	0.1895 (0.057)	0.1554 (0.032)	0.1627 (0.050)	0.1485 (0.040)
	90 %	0.2896 (0.055)	0.2389 (0.077)	0.1992 (0.042)	0.2118 (0.065)	0.1860 (0.049)

TABLE 20

LENGTH OF C.I FOR P{D > t} (EXPONENTIAL MODEL)
$N = 50,\ \theta = 2/3,\ \rho_1 = 1.0,\ \rho_2 = 10.0$

time	C.I (%)	BIN	NOR	MLE	J.K	B.T
0.5	80 %	0.1719 (0.012)	0.1601 (0.012)	0.0855 (0.008)	0.1873 (0.074)	0.1645 (0.044)
	90 %	0.2160 (0.015)	0.2056 (0.016)	0.1098 (0.010)	0.2482 (0.098)	0.2158 (0.056)
1.0	80 %	0.1962 (0.003)	0.1790 (0.004)	0.1223 (0.006)	0.1523 (0.042)	0.1479 (0.023)
	90 %	0.2448 (0.004)	0.2298 (0.005)	0.1571 (0.007)	0.2019 (0.056)	0.1897 (0.031)
1.5	80 %	0.1911 (0.007)	0.1737 (0.007)	0.1320 (0.002)	0.1542 (0.043)	0.1474 (0.019)
	90 %	0.2383 (0.009)	0.2231 (0.009)	0.1694 (0.002)	0.2044 (0.057)	0.1907 (0.025)
2.0	80 %	0.1772 (0.011)	0.1595 (0.012)	0.1269 (0.006)	0.1496 (0.042)	0.1397 (0.019)
	90 %	0.2208 (0.015)	0.2048 (0.016)	0.1629 (0.008)	0.1982 (0.055)	0.1803 (0.024)
3.0	80 %	0.1435 (0.019)	0.1245 (0.020)	0.0996 (0.013)	0.1178 (0.037)	0.1073 (0.022)
	90 %	0.1780 (0.024)	0.1598 (0.026)	0.1279 (0.017)	0.1561 (0.049)	0.1372 (0.028)
4.0	80 %	0.1140 (0.021)	0.0932 (0.024)	0.0701 (0.015)	0.0812 (0.031)	0.0733 (0.021)
	90 %	0.1409 (0.026)	0.1185 (0.032)	0.0900 (0.020)	0.1076 (0.041)	0.0934 (0.028)

TABLE 21

LENGTH OF C.I FOR P{D > t} (EXPONENTIAL MODEL)
$N = 20,\ \theta = 2/3,\ \rho_1 = 1.0,\ \rho_2 = 10.0$

time	C.I (%)	BIN	NOR	MLE	J.K	B.T
0.5	80 %	0.2874 (0.030)	0.2472 (0.033)	0.1374 (0.021)	0.3199 (0.230)	0.2984 (0.158)
	90 %	0.3508 (0.037)	0.3167 (0.045)	0.1765 (0.027)	0.4166 (0.300)	0.4154 (0.238)
1.0	80 %	0.3151 (0.012)	0.2771 (0.013)	0.1926 (0.014)	0.2595 (0.098)	0.2520 (0.077)
	90 %	0.3854 (0.015)	0.3560 (0.016)	0.2473 (0.018)	0.3380 (0.127)	0.3355 (0.121)
1.5	80 %	0.3057 (0.022)	0.2670 (0.024)	0.2046 (0.011)	0.2539 (0.076)	0.2387 (0.056)
	90 %	0.3736 (0.027)	0.3428 (0.031)	0.2627 (0.014)	0.3306 (0.099)	0.3084 (0.078)
2.0	80 %	0.2847 (0.034)	0.2440 (0.038)	0.2947 (0.021)	0.2338 (0.071)	0.2140 (0.051)
	90 %	0.3475 (0.042)	0.3123 (0.051)	0.2499 (0.028)	0.3044 (0.093)	0.2730 (0.066)
3.0	80 %	0.2334 (0.047)	0.1842 (0.061)	0.1515 (0.036)	0.1700 (0.066)	0.1542 (0.054)
	90 %	0.2842 (0.056)	0.2316 (0.081)	0.1940 (0.048)	0.2213 (0.086)	0.1936 (0.066)
4.0	80 %	0.1893 (0.051)	0.1253 (0.075)	0.1069 (0.039)	0.1117 (0.057)	0.1024 (0.049)
	90 %	0.2316 (0.060)	0.1536 (0.096)	0.1351 (0.052)	0.1455 (0.074)	0.1281 (0.060)

TABLE 22

LENGTH OF C.I FOR P{D > t} (HYPOEXP MODEL)
$N = 50, \theta = 0.5, \rho_1 = 2.0, \rho_2 = 10.0, \rho_3 = 2.0$

time	C.I (%)	BIN	NOR	MLE	J.K	B.T
0.5	80 %	0.1417 (0.018)	0.1217 (0.019)	0.0692 (0.006)	0.0813 (0.035)	0.0722 (0.018)
	90 %	0.1758 (0.022)	0.1574 (0.024)	0.0888 (0.008)	0.1077 (0.047)	0.0931 (0.022)
1.0	80 %	0.1855 (0.009)	0.1680 (0.010)	0.1078 (0.007)	0.1110 (0.032)	0.1024 (0.014)
	90 %	0.2313 (0.012)	0.2157 (0.013)	0.1384 (0.009)	0.1472 (0.042)	0.1317 (0.019)
1.5	80 %	0.1964 (0.003)	0.1792 (0.003)	0.1265 (0.004)	0.1344 (0.035)	0.1245 (0.016)
	90 %	0.2451 (0.004)	0.2301 (0.004)	0.1625 (0.006)	0.1182 (0.047)	0.1597 (0.021)
2.0	80 %	0.1920 (0.006)	0.1747 (0.006)	0.1322 (0.002)	0.1413 (0.036)	0.1308 (0.017)
	90 %	0.2395 (0.008)	0.2243 (0.008)	0.1698 (0.002)	0.1872 (0.048)	0.1676 (0.021)
3.0	80 %	0.1675 (0.014)	0.1495 (0.015)	0.1223 (0.007)	0.1265 (0.032)	0.1174 (0.019)
	90 %	0.2185 (0.018)	0.1919 (0.015)	0.1573 (0.009)	0.1677 (0.043)	0.1496 (0.022)
4.0	80 %	0.1390 (0.018)	0.1198 (0.019)	0.1009 (0.011)	0.0988 (0.028)	0.0919 (0.019)
	90 %	0.1723 (0.023)	0.1537 (0.025)	0.1296 (0.015)	0.1310 (0.037)	0.1168 (0.023)

TABLE 23

LENGTH OF C.I FOR P{D > t} (HYPOEXP MODEL)
$N = 20, \theta = 0.5, \rho_1 = 2.0, \rho_2 = 10.0, \rho_3 = 2.0$

time	C.I (%)	BIN	NOR	MLE	J.K	B.T
0.5	80 %	0.2328 (0.044)	0.1839 (0.057)	0.1108 (0.016)	0.1260 (0.051)	0.1177 (0.044)
	90 %	0.2836 (0.053)	0.2314 (0.077)	0.1423 (0.021)	0.1641 (0.067)	0.1592 (0.065)
1.0	80 %	0.3005 (0.023)	0.2614 (0.026)	0.1710 (0.016)	0.1736 (0.050)	0.1633 (0.032)
	90 %	0.3671 (0.029)	0.3354 (0.034)	0.2196 (0.021)	0.2261 (0.066)	0.2105 (0.043)
1.5	80 %	0.3163 (0.011)	0.2785 (0.011)	0.1990 (0.010)	0.2110 (0.053)	0.1954 (0.031)
	90 %	0.3869 (0.013)	0.3576 (0.015)	0.2556 (0.012)	0.2747 (0.069)	0.2502 (0.040)
2.0	80 %	0.3104 (0.016)	0.2722 (0.017)	0.2067 (0.006)	0.2195 (0.050)	0.2017 (0.032)
	90 %	0.3796 (0.020)	0.3495 (0.023)	0.2654 (0.007)	0.2859 (0.065)	0.2571 (0.038)
3.0	80 %	0.2709 (0.035)	0.2289 (0.040)	0.1896 (0.020)	0.1917 (0.047)	0.1756 (0.037)
	90 %	0.3304 (0.043)	0.2924 (0.054)	0.2434 (0.025)	0.2497 (0.062)	0.2211 (0.045)
4.0	80 %	0.2276 (0.046)	0.1771 (0.061)	0.1560 (0.029)	0.1470 (0.047)	0.1354 (0.040)
	90 %	0.2774 (0.055)	0.2223 (0.082)	0.2001 (0.038)	0.1914 (0.061)	0.1696 (0.050)

TABLE 24

LENGTH OF C.I FOR P{D > t} (HYPOEXP MODEL)
$N = 50, \theta = 2/3, \rho_1 = 2.0, \rho_2 = 10.0, \rho_3 = 2.0$

time	C.I (%)	BIN	NOR	MLE	J.K	B.T
0.5	80 %	0.1560 (0.015)	0.1375 (0.016)	0.0855 (0.007)	0.1349 (0.050)	0.1237 (0.034)
	90 %	0.1938 (0.019)	0.1766 (0.020)	0.1098 (0.009)	0.1788 (0.066)	0.1624 (0.045)
1.0	80 %	0.1943 (0.005)	0.1771 (0.005)	0.1226 (0.005)	0.1330 (0.036)	0.1257 (0.017)
	90 %	0.2425 (0.006)	0.2274 (0.006)	0.1574 (0.006)	0.1763 (0.048)	0.1615 (0.021)
1.5	80 %	0.1921 (0.006)	0.1748 (0.007)	0.1324 (0.001)	0.1435 (0.038)	0.1346 (0.014)
	90 %	0.2397 (0.008)	0.2245 (0.008)	0.1700 (0.002)	0.1901 (0.050)	0.1722 (0.019)
2.0	80 %	0.1747 (0.013)	0.1569 (0.014)	0.1273 (0.005)	0.1351 (0.035)	0.1257 (0.015)
	90 %	0.2176 (0.017)	0.2015 (0.018)	0.1634 (0.006)	0.1791 (0.046)	0.1606 (0.020)
3.0	80 %	0.1327 (0.019)	0.1132 (0.021)	0.0998 (0.011)	0.0957 (0.028)	0.0885 (0.018)
	90 %	0.1644 (0.024)	0.1451 (0.028)	0.1281 (0.014)	0.1268 (0.037)	0.1123 (0.022)
4.0	80 %	0.0997 (0.022)	0.0769 (0.026)	0.0700 (0.013)	0.0558 (0.022)	0.0544 (0.016)
	90 %	0.1230 (0.027)	0.0963 (0.036)	0.0898 (0.016)	0.0779 (0.029)	0.0690 (0.021)

TABLE 25

LENGTH OF C.I FOR P{D > t} (HYPOEXP MODEL)
$N = 20$, $\vartheta = 2/3$, $\rho_1 = 2.0$, $\rho_2 = 10.0$, $\rho_3 = 2.0$

time	C.I (%)	BIN	NOR	MLE	J.K	B.T
0.5	80 %	0.2554 (0.040)	0.2112 (0.047)	0.1370 (0.018)	0.2449 (0.197)	0.2360 (0.138)
	90 %	0.3113 (0.049)	0.2683 (0.065)	0.1759 (0.023)	0.3189 (0.257)	0.3274 (0.192)
1.0	80 %	0.3126 (0.015)	0.2746 (0.016)	0.1935 (0.011)	0.2194 (0.076)	0.2105 (0.056)
	90 %	0.3823 (0.019)	0.3525 (0.021)	0.2484 (0.015)	0.2857 (0.098)	0.2744 (0.080)
1.5	80 %	0.3084 (0.020)	0.2699 (0.022)	0.2066 (0.007)	0.2298 (0.060)	0.2134 (0.040)
	90 %	0.3770 (0.025)	0.3465 (0.030)	0.2653 (0.009)	0.2993 (0.078)	0.2716 (0.050)
2.0	80 %	0.2799 (0.039)	0.2381 (0.047)	0.1971 (0.017)	0.2095 (0.057)	0.1915 (0.042)
	90 %	0.3417 (0.048)	0.3044 (0.062)	0.2530 (0.022)	0.2728 (0.075)	0.2403 (0.051)
3.0	80 %	0.2180 (0.047)	0.1648 (0.064)	0.1533 (0.030)	0.1414 (0.052)	0.1297 (0.043)
	90 %	0.2657 (0.057)	0.2054 (0.085)	0.1965 (0.040)	0.1842 (0.068)	0.1619 (0.054)
4.0	80 %	0.1692 (0.048)	0.0964 (0.074)	0.1076 (0.033)	0.0849 (0.042)	0.0794 (0.037)
	90 %	0.2080 (0.056)	0.1163 (0.092)	0.1361 (0.045)	0.1105 (0.055)	0.0995 (0.046)

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